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by Ilham Saifudin

Power Domination Number On Shackle Operation With Points as Lingkage

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ABSTRAK

Abstract: Power dominating set is a way to determine the minimum possible points in a graph that can dominate the connected points around it, with the minimum number of dominating points. The smallest cardinality of a power dominating set is called a power domination number with the notation $\gamma_p(G)$. To find out the minimum limit of the number of power domination numbers in a graph that using the ceiling function comparison between zero forching with $Z(G)$ notation with the largest degree of a graph. The application of power domination number in daily life, one of them is the placement of LBS (Load Break Switch) on the electrical system that is useful as an electrical circuit breaker. The goal is the number of LBS center points becomes minimal and efficient in its use. The graph to be examined in this study is the graph of Shackle operations from a number of special graphs with dots as a link. The resulting operation graphs are: Shackle operation graph with points as lingkage from Path graph $Shack(P_m, v, n)$, Shackle operation graph from Cycle graph $Shack(C_m, v, n)$, Shackle operation graph from Star graph $Shack(S_m, v, n)$. The method used in this research is axiomatic deductive method in problem solving. Understanding the axiomatic method itself is a research method using the principles of proof of deductive valid in mathematical logic by using theorems that already exist in solving a problem. In this study begins by determining the object of research is the graph of the results of the operation Shackle point. Next determine the cardinality of these graphs. After that, determine the point that has the maximum degree on the graph as the dominator point of Power Domination. Then, check whether the nearest neighbor has two or more degrees and analyze its optimization by using the comparison of the ceiliing function between zero forching with the largest degree of graph. Therefore, it can be determined minimal γ_p and dominated.

◆

A. INTRODUCTION

Mathematics is very fundamental in the development of science and technology. Almost in all branches of science use mathematics in problem solving. For example, in the fields of science and social sciences. In this case, a topic that is often discussed in the branch of mathematics and at the same time became the forerunner to technological development, namely about Graph Theory.

Graph Theory is one of the subject matter of Discrete Mathematics which has long been known and widely applied in various fields (Munir, 2004).. In representing the visual of a graph that is by expressing objects with nodes, dots, circles, points, or vertices, while the relationship between objects is expressed by lines or edges (Harary & Frucht, 1969).

One of the interesting topics in Graph Theory is dominating set. The definition of dominating set is a concept of determining the minimum possible points on a graph by determining points as dominating points so that they can reach the points around them (Slamin, Dafik, & Waspodo, 2018). Dominating set has experienced many developments, such as: locating dominating set, independent dominating set, power dominating set, and others.

This paper will discuss the power dominating set. Power dominating set is a way to determine the minimum possible points in a graph that can

dominate the connected points around it, with the minimum number of dominating points. The smallest cardinality of a power dominating set is called a power domination number with the notation γ_p (G) (Benson et al., 2018). To find out the minimum limit of the number of power domination numbers in a graph that is by using the ceiliing function comparison between zero forcing with Z (G) notation with the largest degree of a graph (Benson et al., 2015). The application of power domination number in daily life, one of them is the placement of LBS (Load Break Switch) on the electrical system that is useful as an electrical circuit breaker. The goal is to make the number of LBS focal points minimal and efficient in their use (Haynes, Hedetniemi, Hedetniemi, & Henning, 2002).

Previous studies have investigated Power dominating sets by (Xu, Kang, Shan, & Zhao, 2006), (Zhao, Kang, & Chang, 2006), (Dorfling & Henning, 2006), dan (Dorbec, Henning, Löwenstein, Montassier, & Raspaud, 2013). The theorem as a reference in this research is **Theorem 1**: for any graph $\gamma_p(G) \geq \left\lceil \frac{Z(G)}{\Delta(G)} \right\rceil$ (Benson et al., 2015). Meanwhile, the graph that will be examined in this research is the Shackle operations result from several special graphs with a point as the link. The operation result graphs are: Shackle operation graph with points as lingkage from Path graph $Shack(P_m, v, n)$, Shackle operation graph from Cycle graph $Shack(C_m, v, n)$, Shackle operation graph from Star graph $Shack(S_m, v, n)$. Thus the title can be obtained namely "Power Domination Number on Shackle Operation Graph".

B. METHODS

The method used in this paper is axiomatic deductive method in solving problems. Understanding the axiomatic method itself is a paper method using the principles of deductive validity proof in mathematical logic by using existing theorems in solving a problem (Lestari, 2015).

In this paper begins by determining the object of the paper that the Shackle point operation result graphs. Next determine the cardinality of these graphs. After that, determine the point that has the maximum degree on the graph as the dominator point of Power Domination. Then, check whether the nearest neighbor has two or more degrees and analyze its optimization by using the comparison of the *ceiliing* function between zero *forcing* with the largest degree of graph. Thus, it can be determined γ_p minimal and dominated.

The graph that will be examined in this research is the Shackle operation result graphs from several special graphs with a point as the link. The operation result graphs are: Shackle operation graph with points as lingkage from Path graph $Shack(P_m, v, n)$, Shackle operation graph from Cycle graph $Shack(C_m, v, n)$, Shackle operation graph from Star graph $Shack(S_m, v, n)$.

C. RESULTS AND DISCUSSION

In the results and discussion will be explained about the power dominating number on the Shackle operation result graphs. The results of this paper are several new theorems about power dominating numbers on the Shackle operation result graphs. The initial step in this paper is to determine the cardinality of the Shackle operation result graphs. Then, the second step is to determine the power dominating number on the Shackle operation result graphs. The operation result graphs are: Shackle operation graph with points as lingkage from Path graph $Shack(P_m, v, n)$, Shackle operation graph from Cycle graph $Shack(C_m, v, n)$, Shackle operation graph from Star graph $Shack(S_m, v, n)$. The following are the results of the paper.

1. Shackle Operation Result Graph From Path Graph

In this section, the power dominations number theorem on the Shackle operation graph with points as lingkage from the Path graph $Shack(P, v, n)$ will be presented, followed by theorem evidence. Then an example is given as a visualization of the truth of the theorem.

◊ **Theorem C.1.** For $m \geq 2$ and $n \geq 1$, the power domination number of the graph $Shack(P_m, v, n)$ is $\gamma_p(Shack(P_m, v, n)) = n - 1$.

Proof.

Shackle operational result graph from Path graph $Shack(P_m, v, n)$ is a graph with set point V

$(Shack(P_m, v, n)) = \{A_i; 1 \le i \le n\} \cup \{x_{ij}; 1 \le i \le n-1; 1 \le j \le m\}$ and the set of sides $E(Shack(P_m, v, n)) = \{A_i x_{ij}; 1 \le i \le n-1; 1 \le j \le m\} \cup \{A_{i+1} x_{ij}; 1 \le i \le n-1; 1 \le j \le m\}$. So $p = |V(Shack(P_m, v, n)| = mn + m + n, q = |E(Shack(P_m, v, n))| = 3mn + m - n - 1$, $|Z(Shack(P_m, v, n))| = mn + 1$ and $|\Delta(Shack(P_m, v, n))| = 2m$.

For $m \ge 2$ and $n \ge 1$ it will be shown that $\gamma_p(Shack(P_m, v, n)) \ge n - 1$ by selecting the set of dominator points association that is $S = \{A_i; 1 \le i \le n\}$. Suppose $\gamma_p(Shack(P_m, v, n)) < n$ by taking $n - 2$, then a node will be observed. As an illustration, consider the graph $Shack(P_m, v, n)$ with $\gamma_p(Shack(P_m, v, n)) = n - 2$. It appears that there are nodes that have more than one neighbor so that the other points are not observed. So the supposition above is wrong then $\gamma_p(Shack(P_m, v, n)) \ge n - 1$.

Next it will be shown that $\gamma_p(Shack(P_m, v, n)) \le n - 1$ by selecting the set of dominator points association from $Shack(P_m, v, n)$ which is $S = \{A_i; 1 \le i \le n\}$ which observes other points, then every node in $(Shack(P_m, v, n))$ is observed by $S = \{A_i; 1 \le i \le n\}$, so $\gamma_p(Shack(P_m, v, n)) \le |S| = n - 1$. Therefore $\gamma_p(Shack(P_m, v, n)) \ge n - 1$ and $\gamma_p(Shack(P_m, v, n)) \le n - 1$, then $\gamma_p(Shack(P_m, v, n)) = n - 1$. The set S contains n elements, namely $S = \{A_i; 1 \le i \le n\}$. Thus the value $\gamma_p(Shack(P_m, v, n)) = n - 1$ is obtained. The following will show an example of graph $Shack(P_6, v, 4)$ can be seen in Figure 1.
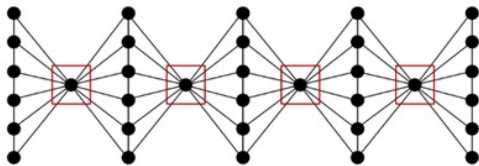

Figure 1 Power Dominating Set Graph $(\boldsymbol{Shack(P_6, v, 4)})$

To strengthen the evidence above, based on **Theorem 1**, namely $\gamma_p(G) \ge \left\lceil \frac{Z(G)}{\Delta(G)} \right\rceil$. It will be shown that $n - 1 \ge \left\lceil \frac{mn+1}{2m} \right\rceil$.

$$\gamma_p(G) \ge \left\lceil \frac{Z(G)}{\Delta(G)} \right\rceil$$

$$n - 1 \ge \left\lceil \frac{mn+1}{2m} \right\rceil$$

$$\leftrightarrow n - 1 \ge \left\lceil \frac{mn}{2m} + \frac{1}{2m} \right\rceil$$

$$\leftrightarrow n - 1 \ge \left\lceil \frac{m}{2} + \frac{1}{2m} \right\rceil$$

Thus $n - 1 \ge \left\lceil \frac{mn+1}{2m} \right\rceil$. ∎

2. Shackle Operation Result Graph from Cycle Graph

In this section, the power dominations number theorem on the Shackle operation graph with points as lingkage from Cycle graph $Shack(C_m, v, n)$ will be presented, followed by theorem evidence. Then an example is given as a visualization of the theorem truth.

◊ **Theorem C.2.** For $m \ge 3$ and $n \ge 1$, the power domination number of the graph $Shack(C_m, v, n)$ is $\gamma_p(Shack(C_m, v, n)) = n - 1$.

Proof.

Shackle operational result graph from Cycle Shack $Shack(C_m, v, n)$ graph is a set of vertices $V(Shack(C_m, v, n)) = \{A_i; 1 \le i \le n\} \cup \{x_{ij}; 1 \le i \le n-1; 1 \le j \le m\}$ and the set of sides $E(Shack(C_m, v, n)) = \{A_i x_{ij}; 1 \le i \le n-1; ; 1 \le j \le m\} \cup \{x_{ij} x_{ij}; 1 \le i \le n-1; 1 \le j \le m\}$. So $p = |V(Shack(C_m, v, n))| = mn + m + n$, $q = |E(Shack(C_m, v, n))| = 3mn + m$, $|Z(Shack(C_m, v, n))| = mn - m + 2$ and $|\Delta(Shack(C_m, v, n))| = 2m$.

For $m \ge 3$ and $m \ge 1$, it will be shown that $\gamma_p(Shack(C_m, v, n)) \ge n - 1$ by selecting the set of dominator points that is $S = \{A_i; 1 \le i \le n\}$. Suppose $\gamma_p(Shack(C_m, v, n)) < n - 1$ by taking $n - 2$, then a node will be observed. As an illustration, consider the graph $Shack(C_m, v, n)$ with $\gamma_p(Shack(C_m, v, n)) = n - 2$. It appears that there are nodes that have more than one neighbor so that the other points are not observed. So the supposition above is wrong then $\gamma_p(Shack(C_m, v, n)) \ge n - 1$.

Next it will be shown that $\gamma_p(Shack(C_m, v, n)) \le n - 1$ by selecting the set of

dominator points from $Shack(C_m, v, n)$ which is $S = \{A_i; 1 \le i \le n\}$ which observes other points, then every node in $(Shack(C_m, v, n))$ is observed $S = \{A_i; 1 \le i \le n\}$, so $\gamma_p(Shack(C_m, v, n)) \le |S| = n - 1$. Therefore $\gamma_p(Shack(C_m, v, n)) \ge n - 1$ and $\gamma_p(Shack(C_m, v, n)) \le n - 1$, then $\gamma_p(Shack(C_m, v, n)) = n - 1$. The set S contains $n - 1$ elements namely $S = \{A_i; 1 \le i \le n\}$. Thus the value $\gamma_p(Shack(C_m, v, n)) = n - 1$ is obtained.

To strengthen the evidence above, based on **Theorem 1**, namely $\gamma_p(G) \ge \left\lceil \frac{Z(G)}{\Delta(G)} \right\rceil$. It will be shown that $n - 1 \ge \left\lceil \frac{mn - m + 2}{2m} \right\rceil$.

$$\gamma_p(G) \ge \left\lceil \frac{Z(G)}{\Delta(G)} \right\rceil$$

$$n - 1 \ge \left\lceil \frac{mn - m + 2}{2m} \right\rceil$$

$$\leftrightarrow n - 1 \ge \left\lceil \frac{mn}{2m} - \frac{m}{2m} + \frac{2}{2m} \right\rceil$$

$$\leftrightarrow n - 1 \ge \left\lceil \frac{n}{2} - \frac{1}{2} + \frac{2}{2m} \right\rceil$$

Thus $n - 1 \ge \left\lceil \frac{mn - m + 2}{2m} \right\rceil$. ∎

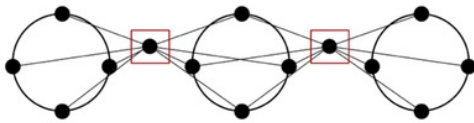The following will show an example of graph $Shack(C_4, v, 2)$ can be seen in Figure 2.

Figure 2 Power Dominating Set Graph
$(Shack(C_4, v, 2))$

3. The Operation Graph from Star Graph

In this section, the power dominations number theorem on the Shackle operation graph with points as lingkage from the Star graph $Shack(S_m, v, n)$ will be presented, followed by theorem evidence. Then an example is given as a visualization of the truth of the theorem.

◊ **Teorema C.3.** For $n \ge 3$ and $n \ge 1$, the power domination number of the graph $Shack(S_m, v, n)$ is $\gamma_p(Shack(S_m, v, n)) = n - 1$.

Bukti.

Shackle operation graph from Star graph $Shack(S_m, v, n)$ is a graph with set point

$V(Shack(S_m, v, n)) = \{A_i; 1 \le i \le n\} \cup \{x_{ij}; 1 \le i \le n - 1; 1 \le j \le m\}$ and the set of sides $E(Shack(S_m, v, n)) = \{A_i x_{ij}; 1 \le i \le n - 1; ; 1 \le j \le m\} \cup \{x_{ij} x_{ij}; 1 \le i \le n - 1; 1 \le j \le m\}$. so $p = |V(Shack(S_m, v, n))| = mn + m + 2n$, $q = |E(Shack(S_m, v, n))| = 3mn + m + 2n$, $|Z(Shack(S_m, v, n))| = mn - m + 2m - 7$ and $|\Delta(Shack(S_m, v, n))| = 2m + 2$.

For $m \ge 3$ and $m \ge 1$, it will be shown that $\gamma_p(Shack(S_m, v, n)) \ge n - 1$ by selecting the set of dominator points namely $S = \{A_i; 1 \le i \le n\}$. Suppose $\gamma_p(Shack(S_m, v, n)) < n - 1$ by taking $n - 2$, then a node will be observed. As an illustration, consider the graph $Shack(S_m, v, n)$ with $\gamma_p(Shack(S_m, v, n)) = n - 2$. It appears that there are nodes that have more than one neighbor so that the other points are not observed. So the supposition above is wrong then $\gamma_p(Shack(S_m, v, n)) \ge n - 1$.

Next, it will be shown that $\gamma_p(Shack(S_m, v, n)) \le n - 1$ by selecting the set of dominator points from $Shack(S_m, v, n)$ yaitu $S = \{A_i; 1 \le i \le n\}$ which observes other points, then every node in $(Shack(S_m, v, n))$ is observed by $S = \{A_i; 1 \le i \le n\}$, so $\gamma_p(Shack(S_m, v, n)) \le |S| = n - 1$. Therefore $\gamma_p(Shack(S_m, v, n)) \ge n - 1$ and $\gamma_p(Shack(S_m, v, n)) \le n - 1$, then $\gamma_p(Shack(S_m, v, n)) = n - 1$. The set S contains $n - 1$ elements namely $S = \{A_i; 1 \le i \le n\}$. Thus the value $\gamma_p(Shack(S_m, v, n)) = n - 1$ is obtained.

To strengthen the evidence above, based on **Theorem 1**, namely $\gamma_p(G) \ge \left\lceil \frac{Z(G)}{\Delta(G)} \right\rceil$. will be shown that $n - 1 \ge \left\lceil \frac{mn + n + 2m - 7}{2m + 2} \right\rceil$.

$$\gamma_p(G) \ge \left\lceil \frac{Z(G)}{\Delta(G)} \right\rceil$$

$$n - 1 \ge \left\lceil \frac{mn + n + 2m - 7}{2m + 2} \right\rceil$$

$$\leftrightarrow n - 1 \ge \left\lceil \frac{n(m + 1) + 2m - 7}{2(m + 1)} \right\rceil$$

$$\leftrightarrow n - 1 \ge \left\lceil \frac{n(m + 1)}{2(m + 1)} + \frac{2m - 7}{2m + 2} \right\rceil$$

$$\leftrightarrow n - 1 \ge \left\lceil \frac{n}{2} + \frac{2m - 7}{2m + 2} \right\rceil$$

Thus $n - 1 \ge \left\lceil \frac{mn + n + 2m - 7}{2m + 2} \right\rceil$. ∎

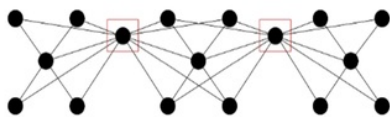The following will be shown an example of graph $Shack(S_4, v, 2)$ can be seen in Figure 3.

Figure 3 Power Dominating Set ($Shack(S_4, v, 2)$)

D. CONCLUSIONS AND SUGGESTIONS

The following are conclusions obtained from the results and previous discussions.

1. Cardinalities of points (orders) and sides (sizes) on Shackle operational result graphs with points as lingkage in this paper are as follows:

   a. $|V(Shack(P_m, v, n))| = mn + m + n$ and $|E(Shack(P_m, v, n))| = 3mn + m - n - 1$;

   b. $|V(Shack(C_m, v, n))| = mn + m + n$ and $|E(Shack(C_m, v, n))| = 3mn + m$;

   c. $|V(Shack(S_m, v, n))| = mn + m + 2n + 1$ and $|E(Shack(S_m, v, n))| = 3mn + m + 2n$.

2. The power domination number on the Shackle operation result with points as lingkage graphs in this paper are as follows:

   a. $\gamma_p(Shack(P_m, v, n)) = n - 1$, for $m \geq 2$ and $n \geq 1$;

   b. $\gamma_p(Shack(C_m, v, n)) = n - 1$, for $m \geq 3$ and $n \geq 1$;

   c. $\gamma_p(Shack(S_m, v, n)) = n - 1$, for $m \geq 3$ and $n \geq 1$;

From the results of the Power Dominating Number research, the researchers provide advice to other researchers to be able to examine the topic with special graphs and other operating results graphs. And can determine applications related to Power Dominating Set to complete solutions in everyday life.

ACKNOWLEDGEMENT

REFERENCES

Benson, K. F., Ferrero, D., Flagg, M., Furst, V., Hogben, L., Vasilevska, V., & Wissman, B. (2015). Power domination and zero forcing, 1–10. Retrieved from http://arxiv.org/abs/1510.02421

Benson, K. F., Ferrero, D., Flagg, M., Furst, V., Hogben, L., Vasilevska, V., & Wissman, B. (2018). Zero forcing and power domination for graph products. *Australasian Journal of Combinatorics*, *70*(2), 221–235.

Dorbec, P., Henning, M. A., Löwenstein, C., Montassier, M., & Raspaud, A. (2013). Generalized power domination in regular graphs. *SIAM Journal on Discrete Mathematics*, *27*(3), 1559–1574. https://doi.org/10.1137/120891356

Dorfling, M., & Henning, M. A. (2006). A note on power domination in grid graphs. *Discrete Applied Mathematics*, *154*(6), 1023–1027. https://doi.org/10.1016/j.dam.2005.08.006

Harary, F., & Frucht, R. (1969). *Graph Theory*. Philippines: Addison-Wesley Publishing Company, Inc.

Haynes, T. W., Hedetniemi, S. M., Hedetniemi, S. T., & Henning, M. A. (2002). Domination in graphs applied to electric power networks. *SIAM Journal on Discrete Mathematics*, *15*(4), 519–529. https://doi.org/10.1137/S0895480100375831

Lestari, K. E. (2015). Analisis Kemampuan Pembuktian Matematis Mahasiswa Menggunakan Pendekatan Induktif-Deduktif Pada Mata Kuliah Analisis Real. *MENDIDIK: Jurnal Kajian Pendidikan Dan Pengajaran*, *1*(2), 128–135.

Munir. (2004). *Matematika Diskrit*. Bandung: Departemen Teknik Informatika Institut Teknologi Bandung.

Slamin, Dafik, & Waspodo, G. A. (2018). Distance Domination Number of Graphs Resulting from Edge Comb Product. *Journal of Physics: Conference Series*, *1022*(1). https://doi.org/10.1088/1742-6596/1022/1/012008

Xu, G., Kang, L., Shan, E., & Zhao, M. (2006). Power domination in block graphs. *Theoretical Computer Science*, *359*(1–3), 299–305. https://doi.org/10.1016/j.tcs.2006.04.011

Zhao, M., Kang, L., & Chang, G. J. (2006). Power domination in graphs. *Discrete Mathematics*, *306*(15), 1812–1816. https://doi.org/10.1016/j.disc.2006.03.037

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8 Submitted to The University of Manchester
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1%

9 Katherine F. Benson, Daniela Ferrero, Mary Flagg, Veronika Furst, Leslie Hogben, Violeta Vasilevska. "Nordhaus–Gaddum problems for power domination", Discrete Applied Mathematics, 2018
Publication

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Internet Source

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23 R LIGRONE. "Development and Liberation of Cauline Gemmae in the MossAulacomnium androgynum(Hedw.) Schwaegr. (Bryales): An Ultrastructural Study", Annals of Botany, 1996
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24 webmail.informatika.org
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25 V. K. Zakharov, A. V. Mikhalev, T. V. Rodionov. "Descriptive Spaces and Proper Classes of Functions", Journal of Mathematical Sciences, 2016
Publication

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