Multiple representations in semiotic reasoning

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Abstract. Representations play an essential role in solving mathematical problems. By using representations, problems that initially seem difficult and complicated can be seen more easily and simply. Semiotics is simply defined as the sign-using to represent a mathematical concept in a problem-solving. In Peirce's theory, semiotic consists of three aspects, namely sign, object, and interpretant. Semiotic reasoning is a process of deducing a conclusion based on the object, sign (representamen), and students interpretation in interpreting the meaning of the sign (interpretant). This paper aims to identify students' mathematical representations of semiotics reasoning in solving fraction problems. This study employed a qualitative approach and used an explorative descriptive type. The participants of the present study were three third grade students. The results show that the first participant used verbal, visual, and symbolic representation. The emerging visual representation was an image. The second participant solved problems by using verbal and visual representations. The visual representation that appears was a diagram. While, the third participant solved problems by using verbal, visual, and symbolic representations. The emerging visual representations were image and diagram. The conclusion is the participants used multiple representations, namely visual, verbal, and symbolic representations in solving problems.

1. Introduction

Representation plays a vital role in mathematical problem-solving activities. Principles and Standards for School Mathematics suggests ten criteria of mathematical literacy and one of which is mathematical representation. NCTM [1] described that students should master the expected standard of representation during teaching and learning activities, that are (1) making and using representation to know, to make a note, and to communicate the mathematical ideas; (2) choosing, applying, and executing translation among mathematical representations to solve a problem; (3) using representation for modeling and interpreting physical phenomena, social, and mathematical phenomena. Many studies have shown that representation support students to achieve mathematical knowledge and lead them to problem-solving skills [2].

The use of several representations in mathematics learning can provide retention of understanding mathematical material and can increase students' interest in mathematics [3]. Students are able to use various mathematical expressions such as visual (graphs, tables, diagrams, and images); symbol (mathematical statement/mathematical notation, numerical or algebraic symbol); and verbal (written words or text) means that they have an ability of multiple mathematical representations [2, 4]. The ability of using multiple representations can provide opportunities for students in solving a problem but in different ways. It will lead students to not only focused on one answer and one representation [5, 6]. In problem-solving, representation of mathematical concepts plays an important role especially in

transforming abstract mathematical ideas into real mathematical concepts [5, 7]. The problem solving process requires reasoning skills.

Reasoning is the ability of students to analyze a problem, solve a problem, draw conclusions and express their ideas logically. According to previous studies, reasoning has been determined as a process of attainment conclusions based on relevant facts and sources [8, 9]. Reasoning can be interpreted as the ability to think logically which is used to obtain conclusions based on statements whose truths have been previously proven [10]. Reasoning related to symbols is called semiotic reasoning.

Semiotics in mathematics is defined as the use of symbols to help an individual in understanding thinking process, representation, and communication [11, 12]. According to Peirce, someone thinks through signs which lead one to communicate with each other and gives any meaning to their environment [13, 14]. The basic principle of Peirce's semiotic theory is everything can be a sign, provided that it has the ability to represent something according to individual interpretation and thinking [15, 16]. Peirce asserts a theory of signs focusing on triad dimensions or trichotomy systems [17]. Moreover, Peirce classifies his theory into three aspects, namely sign, object and interpretant [18-20].

A sign is a representation of an object. An interpretant refers to a notion or notation to represent an object [21]. The sign is only a sign whether it considers a subject as a sign while the interpretant exists if and only if there is a sign. On the other hand, nothing can stands for a sign if there is no interpretant. This relation is an indispensable element of Pierce's semiotic triad. Each sign can act as an object or as an interpreter of another sign [22, 23]. This variety of interpretation forms and representation are involved in the process of reasoning [24]. Therefore, in this study, semiotic reasoning is a process of drawing conclusions based on the objects that have been identified, signs (representamen) made based on objects, and interpretations of signs (interpretant).

In recent years, several researchers have studied representation, reasoning, and semiotics. Minarni et al. [25] elaborated the development of students' mathematical understanding and representation skills through Joyful Problem-based Learning (JPBL). Surya et al. [26] investigated the Junior high school students' visual thinking representation ability in mathematical problem solving by Contextual Teaching and Learning (CTL). Generally, these two previous studies discussed representation in mathematics learning. Meanwhile, other studies discussed semiotic in mathematics learning. Turgut [27] explained a semiotic perspective of students' reasoning on linear transformations when using Dynamic Geometry System (DGS). Sarbo and Yang [21] discussed a semiotic approach to critical reasoning. Brier [28] studied about cyber semiotics and the reasoning powers of the universe. However, the study of mathematical representation in semiotic has never been done by other researchers. Therefore, this study aims to identify the mathematical representation in semiotic reasoning of third-grade students in solving fraction problems.

2. Method

The methodological underpinnings of this study were established through a qualitative approach and an explorative descriptive type. The instruments of this were problem solving task sheets and interview guideline. The prospective participants were 28 students who wherein third-grade students of two state primary schools and one private primary school in Jember. The prospective students were asked to solve non-routine fraction problems individually in 15 minutes. This is in line with the opinion of Brier [28], Batanero, Contreras, and Diaz [29] stated that students should solve individually so that their problem solving skills can be explored well.

The participants of this study were chosen based on the results of problem solving task and interview. They, then, were grouped based on particular characteristics. One participant was elected to represent each group. The data of this study were students' note in solving fraction problems and interview results. The collected data were analyzed to describe students' emerging mathematical representations in semiotic reasoning when they solved fraction problems. The results of the subjects' answers were grouped into three categories of answer. In each category, one subject was taken as a representative subject so that three subjects were obtained

The analysis was carried out through four steps such as (1) data transcription; (2) selecting appropriate data and excluding the unused one; (3) presenting data by grouping the data based on Pierce's semiotic triad namely object, representamen, and interpretant [15, 17, 22]; and (4) drawing a conclusion based on the research findings.

3. Result and Discussion

3.1. The Results of Students' Representation of Semiotic Reasoning in Solving Fraction Problem
The purpose of this study is identifying the mathematical representation in semiotic reasoning of third-grade students in solving fraction problem. The following word problem was given to students:

"Aqla ordered a pizza. She shared half of it to her grandfather. The rest is shared between her three sisters. What is the part of Pizza received by each of Aqla's sisters?"

The emerging semiotic reasoning of first participant (S1) in solving fraction problems were identification of objects, making signs, and interpreting the signs. At the step of identifying objects, representations that appear were verbal and symbolic representations. S1 notes in identifying objects is shown in Figure 1.
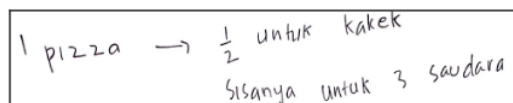

Figure 1. The notes of S1 in identifying objects

Based on Figure 1, it can be seen that S1 used verbal representation in identifying objects when S1 solved the problem. S1 used symbol representation to represent numbers. S1 utilized the symbols 1, ½ and 3 to indicate the number of pizzas and the number of Aqla's sisters. At the step of making signs, S1 drew a picture of a pizza. S1 note in making a sign can be seen in Figure 2.
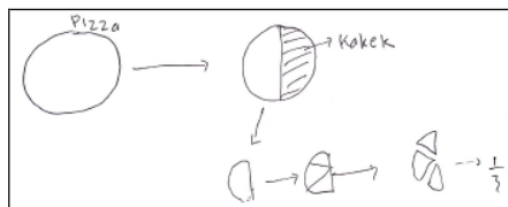

Figure 2. The notes of S1 in making signs

Figure 2 shows that S1 used visual representation. S1 drew a pizza shaped like a circle, then S1 divided the pizza into two parts. One part was shaded to show that the pizza portion was given to grandfather. Another part of the pizza was drawn again and divided into three parts.

Based on the interview result above, data was obtained that S1 made pictures of pizzas so that S1 could understand the intent of the problem easily and divide the portion of pizza for grandfather and three sisters easily. At the step of interpreting signs, S1 used verbal and symbol representations. S1 was accustomed to use symbols to represent numbers. S1 note in interpreting signs can be seen in Figure 3.
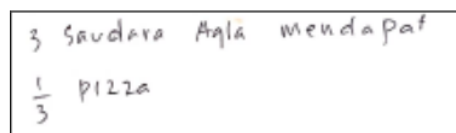

Figure 3. The notes of S1 in interpreting signs

Second participant (S2) employed three steps of semiotic reasoning in solving fraction problems, namely identifying objects, making signs and interpreting signs. However, the emerging representations were different from S1. At the step of identifying objects, S2 used verbal representation. S2 notes in identifying objects is shown in Figure 4.

Figure 4. The notes of S2 in identifying objects

Figure 4 shows that S2 used verbal representations in identifying object when S2 solved the problem. S2 offered the solution by writing the description into brief phrases. Based on the results of interview between the researcher and S2, it was obtained that S2 mentioned sentences which were written on the problem sheet. At the step of making a sign, the emerging representations were are visual and verbal representations. S2 notes in making signs is shown in Figure 5.
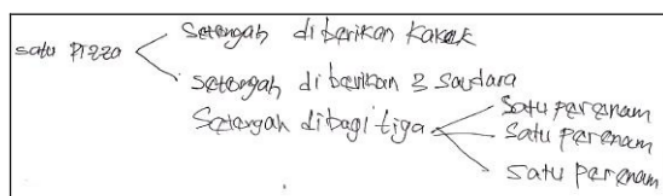

Figure 5. The notes of S2 in making signs

Figure 5 displays the use of verbal representation by S2 in expressing the given information of the problem. At the step of making a sign, the visual representation made by S2 was a diagram. Figure 5 also shows that S2 made a diagram to divide the pizza to grandfather and all sisters. Subsequently, S2 drew a diagram to divide the half rest of the pizza divided into three sisters. At the step of interpreting signs, the representation that appears was verbal representation. By having the verbal ability, S2 could solve the problem correctly. S2 notes in interpreting signs is shown in Figure 6.

Figure 6. The notes of S2 in interpreting signs

Based on the result of S3 in solving fraction problem, it was found that the emerging representations of S3 in semiotic reasoning was different from S1 and S2 had. At the step of identifying objects, S3 used multiple representations, namely verbal and visual representations. The visual representation used by S3 was a diagram. S3 notes in identifying objects can be seen in Figure 7.
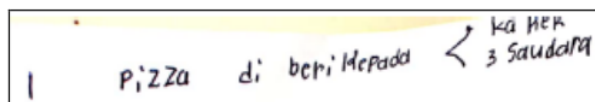

Figure 7. The notes of S3 in identifying objects

At the step of making a sign, the multiple representations emerged again. In the first step of making a sign, S3 drew a picture of a pizza. S3 then made a diagram and drew two half-parts of pizza to

demonstrate how S3 shared the pizza for grandfather and three sisters. After that, S3 drew half of pizza and made a diagram again to share the half part for three sisters. S3 notes in making signs is shown in Figure 8.
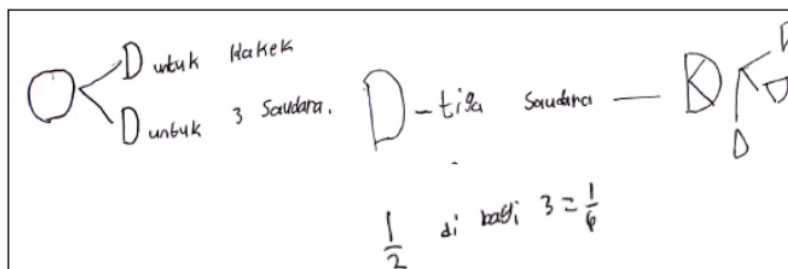

Figure 8. The notes of S3 in making signs

The last step in semiotic reasoning is interpreting signs. In the last step, S3 used multiple representations, namely symbol and verbal representation. Based on the results of interview, S3 was accustomed to use symbol representations to express numbers ½, 1/6 and 6. Moreover, S3 used verbal representation to draw a conclusion. S3 notes in interpreting the signs can be seen in Figure 1.9 as follows.
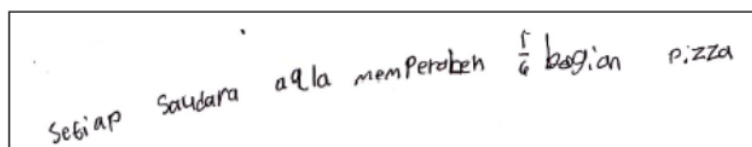

Figure 9. The notes of S3 in interpreting the signs

3.2. Discussion
After analyzing the solutions of three participants in solving fraction problem, the emerging representations from their answers were vary. In the stage of identifying objects, the emerging representations on S1's solution were verbal and symbol representations, while the representation on S2's solution was verbal representation. The emerging representation on S3's solution were verbal and visual representations. It can be inferred that there was one similar representation emerged on the solution of all participants, namely verbal representation. This finding is in line with the results of Bal's study [6] which described that the most widely used representation was verbal representation to understand the problem. In other words, at the step of identifying objects, participants are particularly expected to define the problem use their own sentences which means they understand what the problem is.

The analysis results regarding the step of making a sign show that several representations emerged in the solution of the participants. In the solution of S1, the emerging representation was visual representation. S1 drew an image to represent his answer. This result is supported by the opinion of Eco [30] and Bradbury [31] which stated that one of the signs made by students was an image. An image stands for the properties of the icon as indicated by their own simple nature [32]. Unlike S1, at the step of making a sign, the emerging representation in S2 solution were visual and verbal representations. The verbal representation was used by S2 to express problems in the problem. This finding complements the studies of Madrid, Maz-machado, & Leon-mantero [33]. Their studies discovered that the representations used to express the concept and model were verbal and symbol (numerical) representations. The visual representation was used by S2 to understand the presented problems easily. This finding shows the similarity with the results of the previous studies [34-36] which was about the use of visual representation provide support for the process and transformation of abstract mathematical concepts to concrete concepts. Same as the solution of S2, at the step of making

signs, the emerging representation of S3 solution were verbal and visual representations. The verbal representation was used by S3 to solve non-routine problems. This finding shows parallels with the study of Bal [6]. The visual representation, at the step of making a sign, was used by S3 was a diagram. A diagram is an icon that represents their own relational nature [11, 37]. It can be said that at the step of making, participants mostly used visual representation.

At the step of interpreting signs, the emerging representation on the solution of S2 was verbal representation. While, S1 and S3 emerged multiple representations such as verbal and symbolic representations. In other words, the common representation emerged during interpreting signs was symbolic representation in which participants made a significant meaning to the signs that can be formed as numeric or written text.

Based on the analysis results of the solution of three participants, it can be inferred that multiple representation can emerge on students' activities in solving problems. This is in line with the statement of Bal [6] & Bannister & Flexible [38]. Bal, Bannister & Flexible claimed that the use of multiple representation when solving problems avoided the limitations of one type of representation and constructs new representations. The various representations also provides opportunity for students to bring their ideas up. Ultimately, students can express their ideas when they solve problem by their own way so as they create different presentations [36].

4. Conclusion

The results of this study shows that multiple representations emerged on semiotic reasoning in solving fraction problems process. At the step of identifying objects, the emerging representation were verbal and symbolic representations. The emerging representations, at the step of making signs, were verbal and visual representations. Moreover, the visual representations that are used by participants were images and diagrams. At the step of interpreting signs, the emerging representations were symbolic and verbal representation. In this study, the researchers focused on emerging representations on semiotic reasoning in solving fraction problems. For further research, it is recommended to examine the problem of students' interpretation in semiotic reasoning in solving fractions.

5. Acknowledgments

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