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Local irregularity chromatic number of vertex shackle product of graphs

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Abstract. Local irregular vertex labeling is one of graph labeling type that can be used as a tool for graph coloring. A mapping l is called local irregular vertex labeling if there are: (i) a mapping $l: V(G) \rightarrow \{1, 2, \dots, k\}$ as vertex irregular k -labeling and a weight $w: V(G) \rightarrow \mathbb{N}$, for every $uv \in E(G)$, $w(u) \neq w(v)$ where $w(v) = \sum_{u \in N(v)} l(u)$; and (ii) $opt(l) = \min\{\max\{l_i\}\}$. Thus, the labeling l induces a proper vertex coloring of G where the vertex v is assigned the color $w(v)$. The local irregular chromatic number of G , denoted by $\chi_{lis}(G)$ the minimum cardinality of the largest label over all such local irregular vertex labeling. In this paper, we determine the local irregular chromatic number of a vertex shackle product of graphs. The vertex shackle products of graphs, denoted by $Shack(G, v, k)$, is the graph constructed from k copies of connected graph G and v as the linkage vertex.

1. Introduction

A graph labeling is an assignment of integers to the vertices or edges, or both that first introduced in 1964. That is why labelling is one of interest topic in graph theory because until now there are some kinds of graph labelling. Let G be a simple connected graph finite and undirected graph with vertex set V and edge set E . A labeling in G is a mapping from the set of elements in a graph (vertices, edges, or both) to a set of numbers (usually positive integers) [1]. There are many types of labelings that have been studied such as magic and anti-magic labelling, distance irregular labelling, and many others [2].

A vertex coloring is an assignment of labels or colors to each vertex of a graph such that no edge connects by two identically colored vertices. The most common type of vertex coloring seeks to minimize the number of colors for a given graph. A vertex coloring also called as the chromatic number of a graph, it is the minimum number of colors needed to produce a proper coloring of a graph [3]. In formal definition, the chromatic number denoted by $\chi(G)$ is the smallest positive integer k such that G is vertex k -colorable (and thus G has k -coloring) [4]. One of topic advance of chromatic number is chromatic number local irregular which be introduced by Kristiana *et al.* [5] and written as $\chi_{lis}(G)$. If $G = (V, E)$ be a connected graph, then local irregularity vertex coloring of G is $l: V(G) \rightarrow \{1, 2, \dots, k\}$ and $w: V(G) \rightarrow \mathbb{N}$ where $w(u) = \sum_{v \in N(u)} l(v)$ such that $opt(l) = \min\{\max\{l_i\}\}$ and for every $uv \in E(G)$, $w(u) \neq w(v)$. The result that have been found before about chromatic number



local irregular are chromatic number local irregular of path, cycle, complete, bipartite, star, friendship, and corona product graphs [6]. Figure 1 shows a graph with local chromatic number $\chi_{lis}(G) = 3$.

A shackle product graph is divided into two type. That is vertex shackle and edge shackle product graphs. Both of them are formed from k copies of graph G . A vertex shackle product of graph, denoted by $Shack(G, v, k)$ is the graph constructed from k copies of connected graph G and v as the linkage vertex. Whereas edge shackle products graphs is denoted by $Shack(G, e, k)$ means that the graph is constructed from any G graph of k copies and e as the linkage edge [7]. In this paper we show the local irregular chromatic number of vertex shackle products of graphs. There are some definitions and prepositions that are used in this paper as presented below:

Definition 1. [1] l is called local irregularity vertex coloring if there is a function $l: V(G) \rightarrow \{1, 2, \dots, k\}$ as vertex irregular k -labeling, $w: V(G) \rightarrow N$ where $w(u) = \sum_{v \in N(u)} l(v)$ and

i. $opt(l) = \min\{\max\{l_i\}; l_i, \text{ vertex irregular labeling}\}$

ii. For every $uv \in E(G)$, $w(u) \neq w(v)$.

Definition 2. [1] The minimum cardinality of local irregularity vertex labeling of G is called local irregular chromatic number, written by $\chi_{lis}(G)$

Proposition 1. [1] $opt(l_i) = 1$ if pair of two adjacent vertices in G have different degrees.

Proposition 2. [1] $opt(l_i) \geq 2$ if two adjacent vertices in G have the same degree.

Lemma 1. [1] For every G be simple and connected graph, $\chi_{lis}(G) \geq \chi(G)$

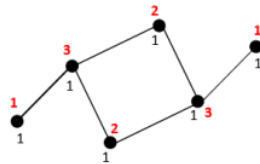


Figure 1. Graph G with $\chi_{lis}(G) = 3$

2. Result

We begin this section with some observations regarding to the chromatic number and local irregularity chromatic number of vertex shackle product of graphs such as path, cycle, star, complete and wheel graph.

Observation 1. The chromatic number of vertex shackle products of path

$$\chi_{lis}(Shack(P_n, v, m)) = \chi_{lis}(P_n)$$

This is due to the definition of vertex shackle graphs that the vertex shackle of k paths with n vertices is also a path with kn vertices. Furthermore, Kristiana *et al.* [1] showed that

$$\chi_{lis}(Shack(P_n, v, m)) = \chi_{lis}(P_n) = \begin{cases} 2, & \text{for } m \times n = 2, 3 \\ 3 & \text{for } m \times n \geq 4, 5, \dots \end{cases}$$

Observation 2. If $Shack(C_n, v, m)$ is the vertex shackle of m cycles C_n for $m \geq 2$ and $n \geq 3$ then

$$\chi(Shack(C_n, v, m)) = \begin{cases} 2, & \text{for } n \text{ even} \\ 3, & \text{for } n \text{ odd} \end{cases}$$

Observation 3. If $Shack(S_n, v, m)$ is the vertex shackle of m stars S_n for $m \geq 2$ and $n \geq 3$, then $\chi(Shack(S_n, v, m)) = 2$

Observation 4. If $Shack(K_n, v, m)$ is the vertex shackle of m complete graphs K_n for $m \geq 2$ and $n \geq 3$, then $\chi(Shack(K_n, v, m)) = n$.

Observation 5. If $Shack(W_n, v, m)$ is the vertex shackle of m wheel W_n for $m \geq 2$ and $n \geq 3$ then

$$\chi(Shack(W_n, v, m)) = \begin{cases} 3, & \text{for } n \text{ even} \\ 4, & \text{for } n \text{ odd} \end{cases}$$

We now ready to determine the **local irregularity chromatic number of vertex shackle of m cycles** as presented in the following theorem.

Theorem 1. If $Shack(C_n, v, m)$ is the vertex shackle of m cycles C_n for $m \geq 2$ and $n \geq 3$ then

$$\chi_{lis}(Shack(C_n, v, m)) = \begin{cases} 3, & \text{for } n \text{ even} \\ 4, & \text{for } n \text{ odd} \end{cases}$$

Proof. Let C_n be a cycle on $n \geq 3$ vertices and $Shack(C_n, v, m)$ be the vertex shackle of m cycles C_n for $m \geq 2$. Then the vertex set of $Shack(C_n, v, m)$ is $V(Shack(C_n, v, m)) = \{a_{i,j}, 1 \leq i \leq n-1, 1 \leq j \leq m, a_{n,m}\}$ and the edge set of $Shack(C_n, v, m)$ is $E(Shack(C_n, v, m)) = \{a_{i,j}a_{i+1,j}, 1 \leq i \leq n-1, 1 \leq j \leq m\} \cup \{a_{n-1,m}a_{n,m}\}$.

By **Proposition 2** we have $opt(l_i(Shack(C_n, v, m))) = 2$. To show the proof we can divide it into two cases below.

Case 1: for n even.

The vertex shackle of cycles $Shack(C_n, v, m)$ has chromatic number $\chi(Shack(C_n, v, m)) = 2$ for n even and $\chi(Shack(C_n, v, m)) = 3$ for n odd. Based on **Lemma 1** the lower bound is $\chi_{lis}(Shack(C_n, v, m)) = 3 \geq \chi(Shack(C_n, v, m)) = 2$. The upper bound shall be proved by defining $l: V(Shack(C_n, v, m)) \rightarrow \{1, 2\}$ using the following formula.

$$l(a_{i,j}) = \begin{cases} 1, & \text{for } 1 \leq i \leq n-1, i \text{ odd} \\ 2, & \text{for } 2 \leq i \leq n-2, i \text{ even} \end{cases}$$

$$l(a_{n,m}) = 1$$

Therefore, $opt(l) = 2$ and the labeling gives vertex-weight as follows:

$$w(a_{i,j}) = \begin{cases} 2, & \text{for } 2 \leq i \leq n-2, i \text{ even} \\ 4, & \text{for } 1 \leq i \leq n-1, i \text{ odd} \\ 8, & \text{for } i = 1 \text{ linkage vertex}, j = 2, 3, \dots \end{cases}$$

$$w(a_{n,m}) = 2$$

We conclude that $|w(V(Shack(C_n, v, m)))| = 3$. Based on **Definition 1** the upper bound is $\chi_{lis}(Shack(C_n, v, m)) \leq 3$. Hence, $\chi_{lis}(Shack(C_n, v, m)) = 3$ for n even.

Case 1: for n odd.

Based on **Lemma 1** the lower bound is $\chi_{lis}(Shack(C_n, v, m)) = 4 \geq \chi(Shack(C_n, v, m)) = 3$ for n odd. The upper bound shall be proved by defining $l: V(Shack(C_n, v, m)) \rightarrow \{1, 2\}$ using some formula below.

$$l(a_{i,j}) = \begin{cases} 1, & \text{for } 1 \leq i \leq n-2, i \text{ odd} \\ 2, & \text{for } 2 \leq i \leq n-1, i \text{ even} \end{cases}$$

$$l(a_{n,m}) = 1$$

Therefore, $opt(l) = 2$ and the labeling gives vertex-weight as follows:

$$w(a_{i,j}) = \begin{cases} 2, & \text{for } 2 \leq i \leq n-3, i \text{ even} \\ 4, & \text{for } 1 \leq i \leq n-2, i \text{ odd} \\ 3, & \text{for } i = n-1, 1 \leq j \leq m \\ 6, & \text{for } i = 1 \text{ linkage vertex}, j = 2, 3, \dots \end{cases}$$

$$w(a_{n,m}) = 3$$

Therefore $|w(V(Shack(C_n, v, m)))| = 4$. Based on **Definition 1**, the upper bound is $\chi_{lis}(Shack(C_n, v, m)) \leq 4$. Hence, $\chi_{lis}(Shack(C_n, v, m)) = 4$ for n odd. This case completes the proof. \square

The following theorem presents the local irregularity chromatic number of vertex shackle of m stars for $m \geq 2$.

Theorem 2. If $Shack(S_n, v, m)$ is the vertex shackle of m star S_n for $m \geq 2$ and $n \geq 3$ then $\chi_{lis}(Shack(S_n, v, m)) = 3$

Proof. Let S_n be a star on $n \geq 3$ vertices and $Shack(S_n, v, m)$ be the vertex shackle of m stars S_n for $m \geq 2$. Then the vertex set of $Shack(S_n, v, m)$ is $V(Shack(S_n, v, m)) = \{a_{i,j}, b_j; 1 \leq i \leq n-1, 1 \leq j \leq m\} \cup \{a_{n,m}\}$ and the edge set of $Shack(S_n, v, m)$ is $E(Shack(S_n, v, m)) = \{a_{i,j}b_j; 1 \leq i \leq n-1, 1 \leq j \leq m\} \cup \{a_{n,m}b_m\}$. Based on **Observation 3**, vertex shackle of stars $Shack(S_n, v, m)$ has $\chi(Shack(S_n, v, m)) = 2$ and by **Lemma 1**, the lower bound is $\chi_{lis}(Shack(S_n, v, m)) = 2 \geq \chi(Shack(S_n, v, m)) = 2$. However, the lower bound cannot be reached. The upper bound shall be proved by defining $l: V(Shack(S_n, v, m)) \rightarrow \{1\}$. By using **Proposition 1**, we know that $opt(l_i)(Shack(S_n, v, m)) = 1$ and the labeling gives vertex-weight as follows:

$$\begin{aligned} w(a_{i,j}) &= 1, \text{ for } 1 \leq i \leq n-1, 1 \leq j \leq m \\ w(a_{i,j}) &= 2, \text{ for } i = n, 1 \leq j \leq m-1, a_{i,j} \text{ is linkage vertex} \\ w(a_{n,m}) &= 1 \\ w(b_j) &= n \end{aligned}$$

Therefore $|w(V(Shack(S_n, v, m)))| = 3$. By **Definition 1** the upper bound is $\chi_{lis}(Shack(S_n, v, m)) \leq 3$. Then it proven that $\chi_{lis}(Shack(S_n, v, m)) = 3$. \square

The local irregularity chromatic number of vertex shackle of m complete graphs for $m \geq 2$ is presented in the following theorem.

Theorem 3. If $Shack(K_n, v, m)$ is the vertex shackle of m complete graphs K_n , for $m \geq 2$ and $n \geq 3$, then $\chi_{lis}(Shack(K_n, v, m)) = n$.

Proof. Let K_n be a complete graph on $n \geq 3$ vertices and $Shack(K_n, v, m)$ be the vertex shackle of m complete graphs K_n for $m \geq 2$. Then the vertex set of $Shack(K_n, v, m)$ is $V(Shack(K_n, v, m)) = \{a_{i,j}; 1 \leq i \leq n-1, 1 \leq j \leq m\} \cup \{a_{n,m}\}$ and the edge set of $Shack(K_n, v, m)$ is $E(Shack(K_n, v, m)) = \{a_{i,j}a_{i,j}; 1 \leq i \leq n-1, 1 \leq j \leq m\} \cup \{a_{i,j}a_{n,m}\}$.

The vertex shackle of m complete graphs $Shack(K_n, v, m)$ has $\chi(Shack(K_n, v, m)) = n$ as **Observation 4**. Based on **Lemma 1**, the lower bound is $\chi_{lis}(Shack(K_n, v, m)) = n \geq \chi(Shack(K_n, v, m)) = n$. The lower bound cannot be reached. The upper bound will be proved by defining $l: V(Shack(K_n, v, m)) \rightarrow \{1, 2, 3, \dots, n; 1 \leq j \leq m\}$. By **Proposition 1**, we can conclude that $opt(l_i)(Shack(K_n, v, m)) = n$ and the labeling gives vertex-weight as much as n different vertex weight. Therefore $|w(V(Shack(K_n, v, m)))| = n$. By **Definition 1**, the upper bound is $\chi_{lis}(Shack(K_n, v, m)) \leq n$. Hence, $\chi_{lis}(Shack(K_n, v, m)) = n$. \square

Finally, we close this section with the **local irregularity chromatic number of vertex shackle of m wheels for $m \geq 2$ as shown below.**

Theorem 4. If $Shack(W_n, v, m)$ is the vertex shackle of m wheels for $m \geq 2$ and $n \geq 3$, then

$$\chi_{lis}(Shack(W_n, v, m)) = \begin{cases} 4, & \text{for } n \text{ even} \\ 5, & \text{for } n \text{ odd} \end{cases}$$

Proof. Let W_n be a wheel on $n \geq 3$ rim vertices and $Shack(W_n, v, m)$ be the vertex shackle of m wheels W_n for $m \geq 2$. Then the vertex set of $Shack(W_n, v, m)$ is $V(Shack(W_n, v, m)) = \{a_{i,j}, b_j, a_{n,m}, 1 \leq i \leq n-1, 1 \leq j \leq m\}$ and the edge set of $Shack(W_n, v, m)$ is $E(Shack(W_n, v, m)) = \{a_{i,j}a_{i+1,j}, a_{i,j}b_j: 1 \leq i \leq n-1, 1 \leq j \leq m\} \cup \{a_{n-1,m}a_{n,m}\} \cup \{b_ma_{n,m}\}$. By **Proposition 2**, we have $opt(l_i)(Shack(W_n, v, m)) = 2$. We divide the proof into two cases below.

Case 1: for n even.

The vertex shackle of wheels $Shack(W_n, v, m)$ has chromatic number $\chi(Shack(W_n, v, m)) = 3$ for n even and $\chi(Shack(W_n, v, m)) = 4$ for n odd. Based on **Lemma 1**, the lower bound is $\chi_{lis}(Shack(W_n, v, m)) = 4 \geq \chi(Shack(W_n, v, m)) = 3$. The upper bound will be proved by defining $l: V(Shack(W_n, v, m)) \rightarrow \{1, 2\}$ using the following formula.

$$\begin{aligned} l(a_{i,j}) &= \begin{cases} 1, & \text{for } 1 \leq i \leq n-1, i \text{ odd}, 1 \leq j \leq m \\ 2, & \text{for } 2 \leq i \leq n-2, i \text{ even}, 1 \leq j \leq m \end{cases} \\ l(a_{n,m}) &= 1 \\ l(b_j) &= 1, \text{ for } 1 \leq j \leq m \end{aligned}$$

Therefore, $opt(l) = 2$ and the labeling gives vertex-weight as follows:

$$\begin{aligned} w(a_{i,j}) &= \begin{cases} 3, & \text{for } 2 \leq i \leq n, i \text{ even}, 1 \leq j \leq m \\ 5, & \text{for } 1 \leq i \leq n-1, i \text{ odd}, 1 \leq j \leq m \\ 10, & \text{for } i = 1 \text{ linkage vertex}, j = 2, 3, \dots \end{cases} \\ w(b_j) &= \frac{n}{2}(1+2), 1 \leq j \leq m \end{aligned}$$

We conclude that $|w(V(Shack(W_n, v, m)))| = 4$. By **Definition 1**, the upper bound is $\chi_{lis}(Shack(W_n, v, m)) \leq 4$. It means $\chi_{lis}(Shack(W_n, v, m)) = 4$ for n even.

Case 2: for n odd.

Based on **Lemma 1** the lower bound is $\chi_{lis}(Shack(W_n, v, m)) = 5 \geq \chi(Shack(W_n, v, m)) = 4$ for n odd. Then the upper bound will be proved by defining $l: V(Shack(W_n, v, m)) \rightarrow \{1, 2\}$ using the following formula.

$$l(a_{i,j}) = \begin{cases} 1, & \text{for } 1 \leq i \leq n-2, i \text{ odd}, 1 \leq j \leq m \\ 2, & \text{for } 2 \leq i \leq n-1, i \text{ even}, 1 \leq j \leq m \end{cases}$$

$$l(a_{n,m}) = 1$$

$$l(b_j) = 1, \text{ for } 1 \leq j \leq m$$

Therefore, $opt(l) = 2$ and the labeling gives vertex-weight as follows:

$$w(a_{i,j}) = \begin{cases} 2, & \text{for } 2 \leq i \leq n-1, i \text{ even}, 1 \leq j \leq m \\ 4, & \text{for } 1 \leq i \leq n-2, i \text{ odd}, 1 \leq j \leq m \\ 3, & \text{for } i = n, i \text{ odd}, 1 \leq j \leq m \\ 10, & \text{for } i = 1 \text{ linkage vertex}, j = 2, 3, \dots \end{cases}$$

$$w(b_j) = \frac{n-1}{2}(1+2) + 12, \text{ for } 1 \leq i \leq n-1, 1 \leq j \leq m$$

We also can conclude that $|w(V(Shack(W_n, v, m)))| = 5$. By **Definition 1**, the upper bound is $\chi_{lis}(Shack(W_n, v, m)) \leq 5$ prove that $\chi_{lis}(Shack(W_n, v, m)) = 5$ for n odd. Therefore, this case completes the proof. Figure 2 shows the example of local irregular chromatic number of the vertex shackle product of W_4 .

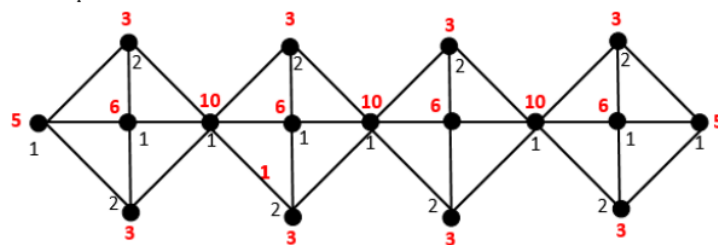


Figure 2. Graph G with $\chi_{lis}(Shack(W_4, v, 4)) = 4$

3. Conclusion

We conclude this paper with an open problem regarding to the local irregular chromatic number of the vertex shackle product of graphs as follows. **Problem 1.** For any connected graph G with n vertices, find the local irregular chromatic number of vertex shackle and edge shackle product of k copies of graph G.

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