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IOP Conference Series: Materials Science and Engineering

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Dominant Local Metric Dimension of Wheel Related Graphs

Dominant Local Metric Dimension of Wheel Related Graphs

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Abstract. Dominant local metric dimension is consist of two interesting topic in graph theory, they are dominating and metric dimension which be expanded as local metric dimension. A graph G is said having dominant local metric dimension if G be a connected graph and there is an ordered subset $W_l = \{w_1, w_2, \dots, w_n\} \subseteq V(G)$ where W_l is a local resolving set as well as a dominating set of G. Minimum cardinality of dominant local resolving set of G is called the dominant local basis of G. Then, cardinality of the dominant local basis of G is called the dominant local metric dimension of G which denoted by $Ddim_l(G)$. In this paper, we determine the dominant local metric dimension of wheel related graphs. That are Wheel, Jahangir, Friendship and Helm graph. Furthermore, we characterize the dominant local metric dimension of those graphs.

Introduction

Graph theory is a branch of discrete mathematics that can be used to solve a problem which can be represented as vertices and edges. Since it was first introduced in 1736 until now, graph theory has led to various new concepts that are always interesting to be researched, such as dominating set and metric dimension. The concept of dominating set developed since the beginning of 1850 [1], this theory studies the existence of a vertex set on a graph that causes every other vertex on the graph to be adjacent to at least one element in the vertex set which be studied and it called the dominating set. The cardinality of the dominating set is called the dominating number. The concept of basis and metric dimensions on a graph was first proposed in ref [2]. A basis on a graph is defined as a vertex set with minimum cardinality which results in each vertex on the graph having a different representation to the basis. Representations are presented using the concept of distance (metric) and the cardinality of basis is called the metric dimension of the graph. The development of the metric dimension concept includes the concept of the strong metric dimension [3], the local metric dimension [4], and dimensions local neighbourliness metric [5]. An application of the metric dimension theory in real life is determine the placement of fire sensors in a building which be written in ref [6].

Based on the explanation above, both the concepts of metric dimensions and dominating set on a graph have the same opportunity to be developed together. Some researchers who have developed this concept include Brigham et al. (2003) who introduced the term resolving domination number, which is a combination of the concept of metric dimensions and dominating set [7]. Next, Henning and

IOP Conf. Series: Materials Science and Engineering 1115 (2021) 012029 doi:10.1088/1757-899X/1115/1/012029

Oellarmann (2004) who developed the same definition but with different term, namely the metric location domination number [8]. A new research in this topic was introduced in ref [9], which determined the dominant metric dimension of some well-known graphs. Then, author combined the concept of the local metric dimension with the concept of dominating set which called the dominant local metric dimension. The concept of the dominant local metric dimension examines the vertex set on a graph which is a minimum local resolving set also as the dominating set of a graph. The results that have been obtained are some characteristics of dominant local metric dimension and its value in special graphs such as path, cycle, complete and bipartite graph [10]. Because the concept is still needed to developed, so this paper will determine the dominant local metric dimension of wheel related graphs. Specific graphs involved in this study include Wheel, Jahangir, Friendship and Helm graphs which described at the following definition below.

Definition 1. [11] Wheel graph W_{n+1} is a graph that contains an *n-cycle* and one additional central vertex that is adjacent to all vertices of the cycle. Wheel graph W_{n+1} has $(n+1)$-vertices and $2n$-*edges*.

Definition 2. [11] Jahangir graph $J_{3,n}$ is a graph with two vertex added between each pair adjacent graph vertices of the outer cycle. The Jahangir graph $J_{3,n}$ has $(3n+1)$-vertices and $4n$-*edges*.

Definition 3. [12] Friendship graph F_n for $n \geq 2$ is a graph constructed by joining n copies of the cycle graph C_3 with a common vertex. This graph has $(2n+1)$-vertices and $3n$-*edges*.

Definition 4. [13] Helm graph H_n is the graph obtained from an *n-wheel graph* by adjoining a pendant edge at each vertex of the cycle. The helm graph H_n has $(2n+1)$-vertices and $3n$-*edges*.

Dominating number of wheel related graphs are given in Table 1.

Table 1. Dominating Number of Wheel Related Graphs [15]-[18]

Graph	Notation	Dominating Number
Wheel	W_{n+1}	$\gamma(W_{n+1}) = 1, n \geq 3$
Jahangir	$J_{3,n}$	$\gamma(J_{3,n}) = n, n \geq 2$
Friendship	F_n	$\gamma(F_n) = n, n \geq 2$
Helm	H_n	$\gamma(H_n) = n, \geq 2$

The following definition and lemma are used to analyse the dominant local metric dimension of wheel related graphs. Some Lemma provides the property of local resolving set of a graph.

Definition 5. [10] Given a connected graph G. An ordered set $W_l = \{w_1, w_2, \ldots, w_n\} \subseteq V(G)$ is called a dominant local resolving set if W_l is local resolving set and dominating set of G. Dominant local resolving set with minimum cardinality is called dominant local basis. Number of vertex in dominant local basis of G is called dominant local metric dimension and denoted by $Ddim_l(G)$.

Lemma 6. [10] Let G be a connected graph and $S \subseteq V(G)$. Every set S containing local resolving set is local resolving set.

Lemma 7. [10] Let G be a connected graph and $W_l \subseteq V(G)$. For every $v_i, v_j \in W_l$ then $r(v_i|W_l) \neq r(v_j|W_l)$.

Lemma 8. [10] Let G be a connected graph, then

$$max\{\gamma(G), dim_l(G)\} \leq Ddim_l(G) \leq min\{\gamma(G) + dim_l(G), |V(G)| - 1\}.$$

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Theorem 9. [14] Let F_n be a friendship graph, then for $n \geq 2$ $dim_l(F_n) = n$.

2. Methodology

This topic consist of some method or prosedure to get the dominant local metric dimension of some graphs related to wheel.

1. Study the definion of dominant local metric dimension.
2. Study some characteristic of dominant local metric dimension.
3. Determine vertex and edge set of the graphs (Wheel, Jahangir, Frienship and Helm graphs).
4. Observe the number of dominating set and local metric dimension of the graphs by this procedure:
    a. Determine the local resolving set of the graphs
    b. Calculating the distance of each vertex with a predetermined, such that every two different vertices adjacent to each other on the graphs has a different representation to the local resolving set.
    c. Determine the local metric dimension and the general pattern of local metric dimensions on the graph.
    d. Determine the dominating set and dominating number.
    e. Compare between the dominating number and local metric dimension
5. Determine the dominant local metric dimension and make proofs by compiling the terms and theorems based on the results obtained.

3. Result and Discussion

This section present the dominant local metric dimension of Wheel, Jahangir, Friendship and Helm graphs. We start by giving a property to show the dominant local metric dimension of wheel graph.

Lemma 10. Let W_{n+1} be a wheel graph of order $n \geq 6$, the edge set is $E(W_{n+1}) = \{uv_i | 1 \leq i \leq n\} \cup \{v_i v_{i+1} | 1 \leq i \leq n-1\} \cup \{v_n v_1\}$, then $W_l = \{v_1, v_4\}$ is local resolving set of W_{n+1}.

Proof. Let $V(W_{n+1}) = \{u, v_i | 1 \leq i \leq n\}$ and $E(W_{n+1}) = \{uv_i | 1 \leq i \leq n\} \cup \{v_i v_{i+1} | 1 \leq i \leq n-1\} \cup \{v_n v_1\}$ with $n \geq 6$. Let $W_l = \{v_1, v_4\}$, for every $v_i \in V(W_{n+1})$ with $i = 1, 2, 3, \dots, n$, then:

$$d(u, v_1) = 1$$
$$d(u, v_4) = 1$$
$$d(v_i, v_1) = \begin{cases} 0 & ; \quad i = 1 \\ 1 & ; \quad i = 2, n \\ 2 & ; \quad i = 3, \dots, n-1 \end{cases}$$
$$d(v_i, v_4) = \begin{cases} 0 & ; \quad i = 4 \\ 1 & ; \quad i = 3,5 \\ 2 & ; \quad i = 6,7, \dots, n, 1,2 \end{cases}$$

Based on the description above, for any two adjacent vertices have different representation. Hence, W_l is local resolving set of W_{n+1} for $n \geq 6$. We can say that if W_{n+1} is a wheel with the order $n \geq 6$ and $E(W_{n+1}) = \{uv_i | 1 \leq i \leq n\} \cup \{v_i v_{i+1} | 1 \leq i \leq n-1\} \cup \{v_n v_1\}$, then $W_l = \{v_1, v_4\}$ is a local resolving set of W_{n+1}.■

Theorem 11. Let W_{n+1} $n \geq 6$, then $Ddim_l\,(W_{n+1}) = \left\lceil \frac{n}{3} \right\rceil$.

Proof. Let W_{n+1} be a wheel graph with order $n \geq 6$, the vertex set of wheel consist of a center vertex and an outer cycle with order n, every vertex in cycle adjacent with a center vertex. Let the vertex set is $V(W_{n+1}) = \{u, v_i | 1 \leq i \leq n\}$ and the edge set is $E(W_{n+1}) = \{uv_i | 1 \leq i \leq n\} \cup \{v_i v_{i+1} | 1 \leq i \leq n-1\} \cup \{v_n v_1\}$, then there exist three possibilities of n.

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1. If $n \equiv 0\ (mod\ 3)$, choose $W_l = \{v_1, v_4, v_7, v_{10}, \dots, v_{3i-2,}, \dots, v_{n-2}\}$ then $|W_l| = \left\lceil \frac{n}{3} \right\rceil$.
2. If $n \equiv 1\ (mod\ 3)$, choose $W_l = \{v_1, v_4, v_7, v_{10}, \dots, v_{3i-2,}, \dots, v_n\}$ then $|W_l| = \left\lceil \frac{n}{3} \right\rceil$.
3. If $n \equiv 2\ (mod\ 3)$, choose $W_l = \{v_1, v_4, v_7, v_{10}, \dots, v_{3i-2,}, \dots, v_{n-1}\}$ then $|W_l| = \left\lceil \frac{n}{3} \right\rceil$.

Because $\{v_1, v_4\} \subseteq W_l$, by Lemma 10, W_l is local resolving set of W_{n+1}. Next, because $E(W_{n+1}) = \{uv_i | 1 \le i \le n\} \cup \{v_i v_{i+1} | 1 \le i \le n-1\} \cup \{v_n v_1\}$, we can see that u adjacent to every vertex of W_l and v_{3i-1} adjacent to v_{3i-2} and v_{3i} adjacent to $v_{3(i+1)-2}$, then W_l is dominating set of W_{n+1}. Hence, W_l is dominant local resolving set of W_{n+1}. Next, we will show that W_l is a dominant local resolving set with minimum cardinality. Choose any $S \subseteq V(W_{n+1})$ with $|S| = |W_l| - 1$, then there exist two possibilities:

a. $u \in S$
If $u \in S$, there exist v_i, v_k with $d(v_i, v_k) > 3$ in cycle such that $v_i, v_{i+1}, v_{i+2}, \dots, v_{k-1}, v_k \notin S$. Therefore $r(v_{i+1}|S) = r(v_{i+2}|S) = (2,2, \dots, 2,1)$, where $v_{i+1}v_{i+2} \in E(W_{n+1})$. Hence, S is not local resolving set of W_{n+1} graph.

b. $u \notin S$
If $u \notin S$ there exist $v_i, v_k \in S$ such that $d(v_i, v_k) = 2$ in cycle. Therefore, S is not dominating set of W_{n+1} graph.

Based on the two possibilities in point a and b, this is contrary with the definition of dominant local metric dimension. Therefore, it is proven that $|W_l| = \left\lceil \frac{n}{3} \right\rceil$ is a dominant local resolving set with minimum cardinality. Thus, for $n \ge 6$ $Ddim_l(W_{n+1}) = \left\lceil \frac{n}{3} \right\rceil$. ■

We now ready to determine the dominant local metric dimension of Jahangir graph. At the first, we show the local resolving set of the graph which is presented in the following Lemma.

Lemma 12. Let $J_{3,n}$ be a Jahangir graph of order $3n+1, n \ge 2$. The edge set is $(J_{3,n}) = \{uv_i; 1 \le i \le n\} \cup \{v_i x_{i1}, x_{i1}x_{i2},; 1 \le i \le n\} \cup \{x_{i2}v_{i+1}; 1 \le i \le n-1\} \cup \{x_{n2}v_1\}$, then $W_l = \{v_1, v_2\}$ is local resolving set of $J_{3,n}$.

Proof. Let $V(J_{3,n}) = \{u, v_i, x_{i1}, x_{i2}; 1 \le i \le n\}$ and $E(J_{3,n}) = \{uv_i; 1 \le i \le n\} \cup \{v_i x_{i1}, x_{i1}x_{i2},; 1 \le i \le n\} \cup \{x_{i2}v_{i+1}; 1 \le i \le n-1\} \cup \{x_{n2}v_1\}$ with $n \ge 2$. Put $W_l = \{v_1, v_2\}$, for every $u, v_i, x_{i1}, x_{i2} \in V(J_{3,n})$ with $i = 1, 2, 3, \dots, n$, then:

$$d(u, v_1) = 1$$
$$d(u, v_2) = 1$$
$$d(v_i, v_1) = \begin{cases} 0 \ ; & i = 1 \\ 2 \ ; & 2 \le i \le n \end{cases}$$
$$d(v_i, v_2) = \begin{cases} 0 \ ; & i = 2 \\ 2 \ ; & i = 3,4, \dots, n-1, n, 1 \end{cases}$$
$$d(x_{i1}, v_1) = \begin{cases} 1 \ ; & i = 1 \\ 2 \ ; & i = n \\ 3 \ ; & 2 \le i \le n-1 \end{cases}$$
$$d(x_{i1}, v_2) = \begin{cases} 1 \ ; & i = 2 \\ 2 \ ; & i = 1 \\ 3 \ ; & 3 \le i \le n \end{cases}$$
$$d(x_{i2}, v_1) = \begin{cases} 1 \ ; & i = n \\ 2 \ ; & i = 1 \\ 3 \ ; & 2 \le i \le n-1 \end{cases}$$
$$d(x_{i2}, v_2) = \begin{cases} 1 \ ; & i = 1 \\ 2 \ ; & i = 2 \\ 3 \ ; & 3 \le i \le n \end{cases}$$

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Based on the description above, for any two adjacent vertices have different representation. Hence, W_l is local resolving set of $J_{3,n}$ for $n \geq 2$. We can say that if $J_{3,n}$ is a Jahangir with the order $3n+1$ for $n \geq 2$ and the edge set is $(J_{3,n}) = \{uv_i; 1 \leq i \leq n\} \cup \{v_i x_{i1}, x_{i1}x_{i2}, ; 1 \leq i \leq n\} \cup \{x_{i2}v_{i+1}; 1 \leq i \leq n-1\} \cup \{x_{n2}v_1\}$, then $W_l = \{v_1, v_2\}$ is a local resolving set of $J_{3,n}$.■

Theorem 13. Let $n \geq 2$, then $Ddim_l(J_{3,n}) = \gamma(J_{3,n})$.

Proof. Let $V(J_{3,n}) = \{u, v_i, x_{i1}, x_{i2}; 1 \leq i \leq n\}$ and $E(J_{3,n}) = \{uv_i; 1 \leq i \leq n\} \cup \{v_i x_{i1}, x_{i1}x_{i2}, ; 1 \leq i \leq n\} \cup \{x_{i2}v_{i+1}; 1 \leq i \leq n-1\} \cup \{x_{n2}v_1\}$. Put $W_l = \{v_1, v_2, \dots, v_n\}$ then $|W_l| = n$. Because $\{v_1, v_2\} \subseteq W_l$, based on the Lemma 12, we get that W_l is a local resolving set of $J_{3,n}$. Next, because $(J_{3,n}) = \{uv_i; 1 \leq i \leq n\} \cup \{v_i x_{i1}, x_{i1}x_{i2}, ; 1 \leq i \leq n\} \cup \{x_{i2}v_{i+1}; 1 \leq i \leq n-1\} \cup \{x_{n2}v_1\}$, we can see that u is adjacent to v_i, v_i adjacent to x_{i1} for $1 \leq i \leq n$ and v_{i+1} adjacent to x_{i2} for $1 \leq i \leq n-1$. Hence, W_l is a dominating set of $J_{3,n}$ and W_l is a dominant local resolving set of $J_{3,n}$. Based on the Table 1, $\gamma(J_{3,n}) = n$, then it can be conclude that W_l is a dominant local resolving set with minimum cardinality and $Ddim_l(J_{3,n}) = \gamma(J_{3,n}), n \geq 2$. ■

The following theorem presents the dominant local metric dimension of a Friendship graph of order $2n+1$ for $n \geq 2$

Lemma 14. Let F_n be a Friendship graph of order $2n+1, n \geq 2$. The edge set is $E(F_n) = \{uv_i, ux_i, v_i x_i; 1 \leq i \leq n\}$, then $W_l = \{v_1, v_2\}$ is local resolving set of F_n.
Proof. Let $V(F_n) = \{u, v_i, x_i; 1 \leq i \leq n\}$ and $E(F_n) = \{uv_i, ux_i, v_i x_i; 1 \leq i \leq n\}$ with $n \geq 2$. Put $W_l = \{v_1, v_2\}$, then for every $u, v_i, x_i \in V(F_n)$ with $i = 1, 2, 3, \dots, n$, then:

$$d(u, v_1) = 1$$
$$d(u, v_2) = 1$$
$$d(v_i, v_1) = \begin{cases} 0 & ; \quad i = 1 \\ 2 & ; \quad 2 \leq i \leq n \end{cases}$$
$$d(v_i, v_2) = \begin{cases} 0 & ; \quad i = 2 \\ 2 & ; \quad i = 3, 4, \dots, n-1, n, 1 \end{cases}$$
$$d(x_i, v_1) = \begin{cases} 1 & ; \quad i = 1 \\ 2 & ; \quad i = 2, 3, \dots, n \end{cases}$$
$$d(x_i, v_2) = \begin{cases} 1 & ; \quad i = 2 \\ 2 & ; \quad i = 3, 4, \dots, n-1, n, 1 \end{cases}$$

Based on the description above, for any two adjacent vertices have different representation. Hence, W_l is local resolving set of F_n for $n \geq 2$. We can say that if F_n is a Friendship with the order $2n+1$ for $n \geq 2$ and $E(F_n) = \{uv_i, ux_i, v_i x_i; 1 \leq i \leq n\}$, then $W_l = \{v_1, v_2\}$ is a local resolving set of F_n.■

Theorem 15. Let $n \geq 2$, then $Ddim_l(F_n) = dim_l(F_n)$.

Proof. Let $V(F_n) = \{u, v_i, x_i; 1 \leq i \leq n\}$ and $E(F_n) = \{uv_i, ux_i, v_i x_i; 1 \leq i \leq n\}$. Put $W_l = \{v_1, v_2, \dots, v_n\}$ then $|W_l| = n$. Because $\{v_1, v_2\} \subseteq W_l$, based on Lemma 14, we get that W_l is a local resolving set of F_n. Next, because $E(F_n) = \{uv_i, ux_i, v_i x_i; 1 \leq i \leq n\}$ we can see that v_i is adjacent to x_i and u. Hence, W_l is a dominating set of F_n and W_l is a dominant local resolving set of F_n. Based on Theorem 9, $dim_l(F_n) = n$, then it can be conclude that W_l is a dominant local resolving set with minimum cardinality and $Ddim_l(F_n) = dim_l(F_n), n \geq 2$. ■

Theorem 16. Let $n \geq 3$, then $Ddim_l(H_n) = \gamma(H_n)$.
Proof. Let $V(H_n) = \{u, v_i, x_i; 1 \leq i \leq n\}$ and $E(H_n) = \{uv_i; 1 \leq i \leq n\} \cup \{v_i x_i; 1 \leq i \leq n\} \cup \{v_i v_{i+1}; 1 \leq i \leq n-1\} \cup \{v_n v_1\}$. Based on the Table 1, we have $\gamma(H_n) = n$. Let $|W_l| = n$,

choose $W_l = \{v_i; 1 \le i \le n\}$. We can see that $d(v_i, u) = d(v_i, x_i) = 1$, then W_l is dominating set of H_n. Without loss of generality, representation of each vertex to W_l can be written as follows.

a. Representation of vertex u to W_l is $r(u|W_l) = (1,1,1,\dots,1)$.
b. Representation of vertex v_i to W_l: $r(v_1|W_l) = (0,1,1,\dots,1)$, $r(v_2|W_l) = (1,0,1,\dots,1)$, ..., $r(v_n|W_l) = (1,1,1,\dots,0)$.
c. Representation of vertex x_i to W_l: $r(u_1|W_l) = (1,2,3,3,\dots,3,2)$, $r(u_2|W_l) = (2,1,2,3,\dots,3,3)$, $r(u_3|W_l) = (3,2,1,2,3,\dots,3)$, …, $r(u_n|W_l) = (2,3,\dots,3,2,1)$.

Representation of each vertex to W_l is different and W_l is dominating set, hence W_l is dominant local resolving set of H_n. Because on the Table 1 we know that dominating number of Helm graph is $(H_n) = n$, then $W_l = n$ is local resolving set with minimum cardinality. So, it is proven that $Ddim_l(H_n) = \gamma(H_n)$. ■

4. Conclusion

Because this topic still new and needed to expand, so we conclude this paper by giving an open problem: determine the dominant metric dimension of some product graphs such as join, corona, or comb product of graphs.

References

[1] Chartrand G, Eroh L, Johnson M A and Oellermann O R 2000 Resolvability in Graphs and The Metric Dimension of A Graph. *Discrete Appl. Math*. **105** 99.
[2] Saputro S W, Mardiana N and Purwasih I A 2013 The metric dimension of comb product graphs. *Graph Theory Conference in honor of Egawa's 60th birthday September 10 to 14* **1-2**
[3] Oellarman and Peters-Fransen 2007 The strong metric dimension of graphs and digraphs *Discrete Applied Mathematics* **155** 356.
[4] Okamoto F,Crosse L,Phinezy B, Zhang P and Kalamazo 2010 The Local Metric Dimension of Graphs *Mathematica Bohemica* **135** 239.
[5] Rodriguez-Velazquez J A, Gomes C G and Ramirez G A B 2014 Computing the local metric dimension of a graph from the local metric dimension of primary subgraphs *International Journal of Computer Mathematics* **92** 1.
[6] Wahyudi S 2018 Aplikasi Dimensi Metrik untuk Meminimalkan Pemasangan Sensor Kebakaran Sebuah Gedung *Jurnal Mathematics and Its Application (Limits)* **15** 89.
[7] Brigham R C, Chartrand G, Dutton R D and Zhang P 2003 Resolving domination in graphs. *Mathematica Bohemica* **128** 25.
[8] Henning M A and Oellermann O R 2004 Metric-locating dominating sets in graphs *Ars Combinatoria -Waterloo then Winnipeg* **73** 1,
[9] Susilowati L, Sa'adah I, Fauziyyah R Z, Erfanian A and Slamin 2020 The Dominant Metric Dimension of Graph *Heliyon* **6** 1.
[10] Umilasari R, Susilowati L and Slamin 2020 the Dominant Local Metric Dimension of Graphs *CAUCHY* **4** 125.
[11] Aytaç A, Odabaş Z N 2011 Residual Closeness of Wheels and Related Networks. Int. J. Found. Comput. Sci. **22** 1229
[12] Gallian J A 2019 A Dynamic Survey of Graph Labelling *The Electronic Journal of Combinatoric* **DS6** 4.
[13] Javaid I, Shokat S 2008 On the Partition Dimension of Some Wheel Related Graphs *Journal of Prime Research in Mathematics* **4** 154.
[14] Cahyabudi A N and Kusmayadi T A 2017 On the local metric dimension of a lollipop graph, a web graph, and a friendship graph *Journal of Physics: Conference Series* **909** 012039
[15] Snyder K 2011 *c-Dominating Sets for Families of Graphs* Fredericksburg: University of Mary Washington.

[16] Mojdeh D A and Ghameshlou A N 2007 Domination in Jahangir Graph $J_{2,m}$. *Int. J. Contemp. Math. Sciences* **24** 193.
[17] Ayhan A K 2011 Determination and testing the Domination Numbers of Helm Graph, Web graph and Levi Graph Using MATLAB *Journal of Education and Science* **24** 103.
[18] Nagabhushana C S, Kavitha B N and Chudamani H M 2017 Split and Equitable Domination of Some Special Graph *International Journal of Science Technology & Engineering* **4** 50.
[19] Ariono D, Aryanti P T P, Subagjo S and Wenten I G 2017 The effect of polymer concentration on flux stability of polysulfone membrane *AIP Conference Proceedings* **1788** 030048.

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