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The Fifth International Conference on Combinatorics, Graph Theory, and Network Topology 2021

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The Fifth International Conference on Combinatorics, Graph Theory, and Network Topology 2021

Dafik

Editor in Chief of International Conference on Combinatorics, Graph Theory, and Network Topology 2021
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We gratefully acknowledge the presence of all participants on "The Fifth International Conference on Combinatorics, Graph Theory, and Network Topology (ICCGANT)". It is the 5th International conference was held on 21-22 August 2021 by the CGANT Research Group, University of Jember in collaboration with the Indonesian Combinatorics Society (INACOMBS). The conference welcomed participants from several countries, with a wide and variety of mathematics research of interests, in particular combinatorial research. The mission of this conference is to be an annual international forum where members of society organizations, research students, educators and researchers, writers, physicists, teachers and practitioners from all over the world can meet and exchange ideas to communicate and discuss theoretical and practical knowledge of mathematical research and its applications. The aim of the fifth conference is to present and discuss the latest research that leads to the exchange of new theoretical, analytical and scientific knowledge and to a deeper understanding of the fields of mathematics, application of mathematics, and mathematics education. The topics of this conference have been identified on the ic.cgant.unej.ac.id website.

There were 200 participants in ICCGANT 2021, consist of research students, academics and researchers, scholars, scientist, teachers and practitioners from many countries. The number of paper submitted to this conference 138 articles, and the number of paper sent to the reviewers were 95 articles. Throughout the tough refereeing process, we have successfully selected some papers to be published on IOP Conference Series: Journal of Physics of 71 papers.

Finally, on behalf of the organizing committee, we gratefully acknowledge the support from the University of Jember of the year 2021. We would also like to express our sincere gratitude to all the lovely participants who have engaged in this unforgettable and important scientific annual forum.

Prof. Drs. Dafik, M.Sc., Ph.D

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All conference organisers/editors are required to declare details about their peer review. Therefore, please provide the following information:

- **Type of peer review:** The type of peer review article is Double-blind review where the authors name and affiliation of the paper are hidden. The reviewer independently made some suggestion and corrections on the papers.

- **Conference submission management system:** All the papers were submitted online through EasyChair system
  https://easychair.org/conferences/?conf=the5thiccgant2021

- **Number of submissions received:** 138

- **Number of submissions sent for review:** 95

- **Number of submissions accepted:**71

- **Acceptance Rate (Number of Submissions Accepted / Number of Submissions Accepted X 100):** 51.45%

- **Average number of reviews per paper:** 3

- **Total number of reviewers involved:** 54

- **Any additional info on review process:** The reviewers gave some feedback to each paper based on the referring guidelines such as the state of the art of research, methodology, a gap, a proposed novelty, originality, and language. They also review the originality of the research finding, and the bibliography cited in their paper. In general they have made refereeing on three things, namely the content, layout, and language.

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We acknowledge that

Reni Umilasari

Muhammadiyah Jember University

has participated as a PAPER PRESENTER

in The 5th International Conference on Combinatorics,
Graph Theory, and Network Topology
entitled

**DETERMINATION OF BULOG REGIONAL SUB-DIVISIONS IN EAST JAVA USING CONNECTED
DOMINATION NUMBER THEORY**

University of Jember - Indonesia
21st - 22nd August 2021

CGANT Research Group
Chairman

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Table of contents

Volume 2157

2022

◂ Previous issue   Next issue ▸

The 5th International Conference on Combinatorics, Graph Theory, and Network Topology (ICCGANT 2021) 21-22 August 2021, Jember, Indonesia

Accepted papers received: 08 December 2021

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Preface

Combinatorics

➕ Open abstract   ▤ View article   📄 PDF

OPEN ACCESS   012003

Implementation of the greedy algorithm on graph coloring

T N Sipayung, S Suwilo, P Gultom and Mardiningsih

+ Open abstract   View article   PDF

OPEN ACCESS   012004

The local strong metric dimension in the join of graphs

R Amalia, Firdausiyah, T Yulianto, Faisol and Kuzairi

+ Open abstract   View article   PDF

OPEN ACCESS   012005

On twin edge mean colorings of graphs

J D Tolentino, M A C Tolentino and E B Bernales

+ Open abstract   View article   PDF

OPEN ACCESS   012006

Some result on integrality of several matrix representation of complete r-uniform hypergraph

A Y Zakiyyah

+ Open abstract   View article   PDF

OPEN ACCESS   012007

On r-dynamic coloring of central vertex join of path, cycle with certain graphs

N Mohanapriya, K Kalaiselvi, V Aparna, Dafik and I H Agustin

+ Open abstract   View article   PDF

OPEN ACCESS   012008

Anonymously tracking covid-19 patient using labeling graph approach and GPS tracking technology

Nuril Lutvi Azizah and Uce Indahyanti

+ Open abstract   View article   PDF

OPEN ACCESS   012009

Determination of bulog regional sub-division in east java using connected domination number theory

Reni Umilasari and Ilham Saifudin

+ Open abstract   View article   PDF

OPEN ACCESS   012010

The mixed metric dimension of wheel-like graphs

Darmaji and N Azahra

✚ Open abstract   ▤ View article   🗎 PDF

OPEN ACCESS   012011

Graphs with strong 3-rainbow index equals 2

A N M Salman, Z Y Awanis and S W Saputro

✚ Open abstract   ▤ View article   🗎 PDF

OPEN ACCESS   012012

On resolving efficient domination number of path and comb product of special graph

I Kusumawardani, Dafik, E Y Kurniawati, I H Agustin and R Alfarisi

✚ Open abstract   ▤ View article   🗎 PDF

OPEN ACCESS   012013

On graceful chromatic number of some graphs

M L Asy'ari, Dafik, I H Agustin, R Nisviasari and R Adawiyah

✚ Open abstract   ▤ View article   🗎 PDF

OPEN ACCESS   012014

On the rainbow antimagic coloring of vertex amalgamation of graphs

J C Joedo, Dafik, A I Kristiana, I H Agustin and R Nisviasari

✚ Open abstract   ▤ View article   🗎 PDF

OPEN ACCESS   012015

On the resolving strong domination number of some wheel related graphs

R Humaizah, Dafik, A I Kristiana, I H Agustin and E Y Kurniawati

✚ Open abstract   ▤ View article   🗎 PDF

OPEN ACCESS   012016

On the r-dynamic chromatic number of subdivision of wheel graph

R Z Riba'ah, Dafik, A.I Kristiana, I.N Maylisa and Slamin

✚ Open abstract   ▤ View article   🗎 PDF

OPEN ACCESS   012017

Improving the robustness of the affine cipher by using a rainbow antimagic coloring

R Nisviasari, Dafik, I H Agustin, E Y Kurniawati, I N Maylisa and B J Septory

❌

+ Open abstract   🗎 View article   📄 PDF

OPEN ACCESS   012018

On local vertex irregular reflexive coloring of graphs

Dafik, D J Koesoemawati, I H Agustin, E Y Kurniawati and R Nisviasari

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OPEN ACCESS   012019

The rainbow vertex antimagic coloring of tree graphs

Marsidi, Ika Hesti Agustin, Dafik, Elsa Yuli Kurniawati and Rosanita Nisviasari

+ Open abstract   🗎 View article   📄 PDF

Applied Mathematics

OPEN ACCESS   012020

Determination of Hue Saturation Value (HSV) color feature in kidney histology image

I Kurniastuti, E N I Yuliati, F Yudianto and T D Wulan

+ Open abstract   🗎 View article   📄 PDF

OPEN ACCESS   012021

Decision-making using fuzzy TOPSIS for selecting beginner UMKM that receive business funding

N L Negari, A Riski, A Pradjaningsih and A Kamsyakawuni

+ Open abstract   🗎 View article   📄 PDF

OPEN ACCESS   012022

H-infinity for autonomous surface vehicle position estimation

T Herlambang, D Rahmalia, A Suryowinoto, F Yudianto, F A Susanto and M Y Anshori

+ Open abstract   🗎 View article   📄 PDF

OPEN ACCESS   012023

Estimation of Packed Red Cells (PRC) in Bojonegoro blood bank using Modified Kalman Filter

A Muhith, T Herlambang, D Rahmalia and D F Karya

+ Open abstract   🗎 View article   📄 PDF

OPEN ACCESS   012024

Estimation of shoulder and elbow joint angle from a linear to quadratic tracking based on six muscle force vectors

T Herlambang, D Rahmalia, H Nurhadi, A Suryowinoto and D F Karya

➕ Open abstract   📄 View article   📄 PDF

OPEN ACCESS   012025

Implementation of the ACS-RVND algorithm on the VRP variant and its application to distribution optimization

Sapti Wahyuningsih, Darmawan Satyananda and Lucky Tri Oktoviana

➕ Open abstract   📄 View article   📄 PDF

OPEN ACCESS   012026

Seawater salinity modeling using bivariate probit regression

Faisol, Tony Yulianto, Arsyiah, Sugiono, Achmad Basuki and Muhammad Agus Zainuddin

➕ Open abstract   📄 View article   📄 PDF

OPEN ACCESS   012027

Profitability estimation of XYZ company using H-infinity and Ensemble Kalman Filter

M Y Anshori, T Herlambang, P Katias, F A Susanto and R R Rasyid

➕ Open abstract   📄 View article   📄 PDF

OPEN ACCESS   012028

Oscillation and Asymptotic Behavior of Second-Order Half-Linear Noncanonical Difference Equations of Advanced Type

P Gopalakrishnan, A Murugesan, Dafik and I H Agustin

➕ Open abstract   📄 View article   📄 PDF

OPEN ACCESS   012029

Estimation of Thrombocyte Concentrate (TC) in PMI Gresik using unscented and square root Ensemble Kalman Filter

A Muhith, T Herlambang, D Rahmalia, Irhamah and D F Karya

➕ Open abstract   📄 View article   📄 PDF

OPEN ACCESS   012030

Hybrid clustering and classification methods to find out the pattern of the spread of covid-19 in East Java province

M W F Umam, M Fatekurohman and D Anggraeni

➕ Open abstract   📄 View article   📄 PDF

                                                                                    012031
                                                                                    ✖

Design of conveyor table with quality function deployment method and statistical analysis of anthropometry data approach as a physical distance tool for SMEs

Zulkani Sinaga, Yuri Delano Regent Montororing and Achmad Muhazir

+ Open abstract   ▤ View article   🗎 PDF

OPEN ACCESS   012032

Production process improvements to minimize product defects using DMAIC six sigma statistical tool and FMEA at PT KAEF

Yuri Delano Regent Montororing, Murwan Widyantoro and Achmad Muhazir

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OPEN ACCESS   012033

n-cutting site of DNA splicing language for single string and palindromic rule

N M Ruslim, Y Yusof and N Adzhar

+ Open abstract   ▤ View article   🗎 PDF

OPEN ACCESS   012034

The quality of coffee bean classification system based on color by using k-nearest neighbor method

Nelly Oktavia Adiwijaya, Hammam Iqomatuddin Romadhon, Januar Adi Putra and Dewangga Putra Kuswanto

+ Open abstract   ▤ View article   🗎 PDF

Education

OPEN ACCESS   012035

Combination of high order kernel estimators for estimation of fruit trees biomass on critical land

Zulfikar, Munawarah and Ambar Susanti

+ Open abstract   ▤ View article   🗎 PDF

OPEN ACCESS   012036

A meta analysis study: is Problem Based Learning (PBL) effective toward students' mathematical connections ability?

S Aisyah and D Usdiyana

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OPEN ACCESS   012037

An analysis of students learning independence in mathematics based on google classroom

T N Sipayung, Imelda, T Y E Siswono and Masriyah

➕ Open abstract   📄 View article   📄 PDF

OPEN ACCESS   012038

High order thinking skills: can it arise when a prospective teacher solves a controversial mathematics problem?

Alfiani Athma Putri Rosyadi, Cholis Sa'dijah, Susiswo and Swasono Rahardjo

➕ Open abstract   📄 View article   📄 PDF

OPEN ACCESS   012039

An illustration of mathematical skills: the procept (process-concept) junior high school students in solving mathematical problems

Prayogo, St Suwarsono and Siti Khabibah

➕ Open abstract   📄 View article   📄 PDF

OPEN ACCESS   012040

The analysis of students metaliteracy under the implementation of RBL-STEM in solving graph rainbow antimagic coloring problems

E Y Kurniawati, Dafik, I H Agustin and I N Maylisa

➕ Open abstract   📄 View article   📄 PDF

OPEN ACCESS   012041

Science process skills students on cells and tissues concept during the covid-19 pandemic: how did it achieve?

H Fitrianingrum and M F Noor

➕ Open abstract   📄 View article   📄 PDF

OPEN ACCESS   012042

Improving students' mathematical intuitive thinking ability using analogy learning model

L Kurniawati, I S Farhana and R Miftah

➕ Open abstract   📄 View article   📄 PDF

OPEN ACCESS   012043

Didactical design of mathematical reasoning on three dimensional in high school

R Miftah, A N Amalina and L Kurniawati

➕ Open abstract   📄 View article   📄 PDF

OPEN ACCESS   012044

The synthesis of qualitative evidence-based learning by design model to improve TPACK of

prospective mathematics teacher

Yurniwati and Erry Utomo

+ Open abstract   📄 View article   📄 PDF

012045

Development of mathematics teaching-learning material with metaphors approach

G Dwirahayu, I D Handayani, O Suhyanto, E Musyrifah and D Sobiruddin

+ Open abstract   📄 View article   📄 PDF

012046

The Science Environment Technology Society (SETS) based e-module development with project based learning model in colloidal learning

W Azura, A Silalahi, M Zubir and Nurfajriani

+ Open abstract   📄 View article   📄 PDF

012047

The usefulness of LabXChange virtual lab and PhyPhox real lab on pendulum student practicum during pandemic

D Nanto, R D Agustina, I Ramadhanti, R P Putra and D Mulhayatiah

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Determination of bulog regional sub-division in east java using connected domination number theory

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Determination of bulog regional sub-division in east java using connected domination number theory

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Abstract. Bulog is a state-owned public company engaged in food logistics. There are about 18 Bulog warehouses in East Java Province with an uneven distribution. Because the scope of Bulog's business includes logistics or warehousing, surveys and pest eradication, provision of plastic bags, transportation business, food commodity trading and retail business, there must be good coordination between regional sub-divisions and warehouse complexes. In this study, the minimum regional sub-division recommendations in the East Java region were determined using the theory of connected domination number in graphs. The location of the warehouse complex is described as the vertex and adjacent warehouse complexes are connected by edges. In this research, we also determine the connected domination number of unicyclic graph and vertex amalgamation operations for several classes of graphs.

1. Introduction

BULOG is a state-owned public company engaged in food logistics. The company's business scope includes logistics/warehousing, surveys and pest control, supply of plastic bags, transportation business, food commodity trading and retail business. As a company that continues to carry out public duties from the government, BULOG continues to carry out activities to maintain the Basic Purchase Price for grain, stabilize prices, especially basic prices, distribute rice for social assistance (Bansos) and manage food stocks [1].

BULOG in an area is tasked with meeting the rice consumption needs not only for its own region, but BULOG is also tasked with supplying other regions with a rice deficit. At Perum BULOG, the distribution of rice stocks is known as "MoveNas" which stands for Move Nasional. Prior to the Move Nas, Perum BULOG will meet stock needs in all warehouses in each region through procurement from local farmers [1]. If the stock in the area has been met, the rice will be moved to other areas that need it to meet the rice stock in the area. This is done so that the distribution of rice throughout Indonesia is even. In order to optimize this role, it is necessary to prepare a strategy in terms of regulating which Bulogs have excess stock, so that other Bulogs which usually require supplies can be observed in order of priority. The distance and access between Bulogs also need to be considered. To observe this, we can use connected dominating number theory. Perum BULOG has more than a thousand warehouse units spread throughout Indonesia which are managed by 26 Regional Offices. These warehouses are used to store rice reserves that have been absorbed by local farmers. Of all Regional Offices, there are 3 regions that have the largest storage capacity. In the first position, the East Java Regional Office has a total of 378 warehouse units with a total capacity of more than one million tons

[1]. The distribution of Bulog's warehouse units can be described as points on the graph, while the easy and close distribution access is analogous to the edges connecting the points. Furthermore, from the graph formed, it can be determined which main warehouses are considered as supplying warehouses for rice stocks for other areas.

Domination is one of the topics discussion in graph theory which is known from the philosophy of the chess game. Dominating Number is one of the interesting topics in graph theory. The dominating number has existed since 1850 [2]. It appeared among chess fans in Europe, namely the determination of how many queens should be placed on an 8×8 chessboard, so that all tiles on the chessboard can be controlled by the queen and the number of queens placed on the board and it must be minimal. The problem of domination is one of the most widely studied topics in graph theory [3]. Several studies on dominating number have been developed, including: On Domination Number of Cartesian Product of Graphs yang diteliti oleh Gravier dan Mollard [4], The Domination Number of The Cartesian Products of Path and Cycles [5]. While in 1997, topics related to dominating number were developed namely connected dominating set [6]. Other research related to the dominating of the graph, such as Split Domination Number of Some Special Graphs [7,10] and Results Connecting Domination, Steiner and Steiner Domination Number of Graphs by Ramalakshmi and K. Palani [8]. The concept of connected dominating set is the suitable concept for Determination of Bulog Regional Sub-Divisions. In 1996 Some inequalities about connected domination number are given by Cheng Bo and Bollian Liu in 1996 [9]. And in 2013, some researcher can determine the Bounds on The Connected Domination Number of a Graph based on the value of girth, minimum degree, and connected order sum number [11]. The formal definition of dominating set and dominating of graph are showed in Definition 1 and Definition 2 as below.

Definition 1. [12] Let G be a graph with vertex set V and edge set E. Let S be the subset of vertex set V. If every vertex in V\S is adjacent to minimum one vertex of S then S is said to be a dominating set.

Definition 2. [12] The size of a smallest dominating set is referred as the dominating number of a graph G denoted by γ(G).

While the main difference of dominating set and connected dominating set is if the connected dominating set must be in the form of subgraph of the main graph like in Definition 3. Figure 1 gives the difference among dominating set and connected dominating set of a graph.

Definition 3. [6] A dominating Set D is said to be connected dominating set, if the induced sub graph D is connected. The connected domination number γ_c (G) is the minimum cardinality of a connected dominating set.
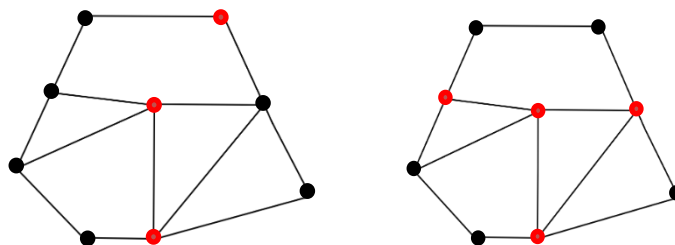
Figure 1. A graph $\boldsymbol{G}$ with dominating number $\boldsymbol{\gamma(G) = 3}$ and connected dominating number $\boldsymbol{\gamma_c(G) = 4}$.

2. Results and Discussion

We start this section by giving the observation concerning on the connected domination number of unicyclic graph such as cycle with two pendant, pan graph, subdivision of pan graph, and subdivision of sun graph.

Observation 1. Let C_n^2 be a cycle with two pendant graph, for $n \geq 3$, $\gamma_c(C_n^2) = \gamma_c(C_n)$.
In Theorem 4 we know that $\gamma_c(C_n) = n - 2$. For C_n^2 graph, it has minimum connected dominating set if two vertices which adjacent to the pendant are elements of connected dominating set, so it can dominate both of the pendants. Thus, we can say that $\gamma_c(C_n^2) = \gamma_c(C_n)$.

Observation 2. Let $\mathcal{P}(n)$ be a pan graph, for $n \geq 3$, $\gamma_c(\mathcal{P}(n)) = \gamma_c(C_n)$.
This case is equivalent with observation 9, the difference is pan graph just hs one pendant. Thus the minimum connected dominating set is $n - 2$ where the vertex whose degree equals three is also element of connected dominating set. Therefore we can say that $\gamma_c(\mathcal{P}(n)) = \gamma_c(C_n)$.
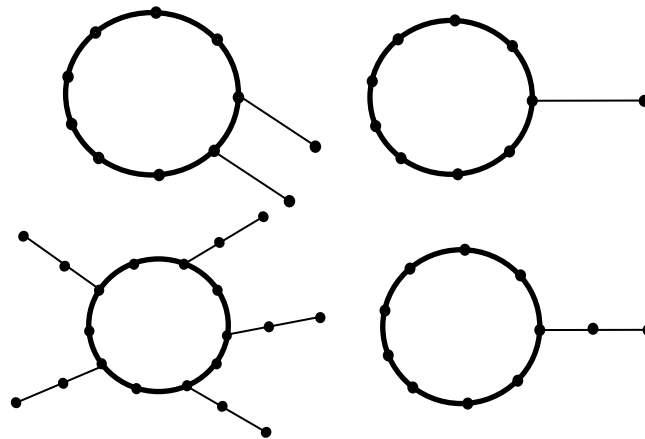
Figure 2. Connected dominating set of unicyclic graph.

Observation 3. Let $S(\mathcal{P}(n))$ be a subdivision of pan graph, for $n \geq 3$, $\gamma_c(S(\mathcal{P}(n))) = 2n - 1$.
Subdivision of pan graph can be said as a cycle graph which one of the vertex has degree equals tree and connected to two vertices (it form a path with length is two). Then the connected dominating set will be minimum if we choose $2n - 2$ vertices in cycle are element of connected dominating set and also a vertex that adjacent to the vertex whose the degree equals tree. So, the connected dominating number is $\gamma_c(S(\mathcal{P}(n))) = 2n - 2 + 1 = 2n - 1$.

Observation 4. Let $S(S(n))$ be a subdivision of sun graph, for $n \geq 3$, $\gamma_c(S(S(n))) = 3n - 1$.
Subdivision of sun graph has n pendant, so all vertices which adjacent to the pendant must be elements of connected dominating set. To make a connected subgraph, so all vertices in the cycle also must be elements of connected dominating set. For getting the minimum cardinality, we can remove one vertex which the degree is two for not to be elements of connected dominating number. Then, $\gamma_c(S(S(n))) = 3n - 1$.

Next, we show the Connected Dominating Number of vertex amalgamation graph that will be describing on Theorem 1 to Theorem 4.

Theorem 1. Let $G = \vee_{\{v\}}^1 \{W_{n_1}, W_{n_2}, \ldots, W_{n_t}\}$ is amalgamation vetex of wheel graph, then
$$\gamma_c(G) = \begin{cases} 1, & v \text{ is centre vertex of } W_{n_i}, 2 \leq i \leq t \\ t + 1, & v \text{ is outer vertex of } W_{n_i}, 2 \leq i \leq t \end{cases}$$
Proof: If the linkage vertex of $G = \vee_{\{v\}}^1 \{W_{n_1}, W_{n_2}, \ldots, W_{n_t}\}$ is centre vertex of W_{n_i} then for any $k \in V(G) \backslash v$ show that $d(v, k) = 1$. It means that v can dominate all vertices of $V(G)$, thus $\gamma_c(G) = 1$. If the linkage vertex of $G = \vee_{\{v\}}^1 \{W_{n_1}, W_{n_2}, \ldots, W_{n_t}\}$ is outer vertex of W_{n_i} then for any k element of outer vertex in every W_{n_i} show that $d(v, k) = 2$. Let S is connected dominating number of G, so we

can take v_i which the centre vertex of every W_{n_i} with $d(v, v_i) = 1$ as the element of S. Then the set $S = \{v, v_i | 2 \le i \le t\}$ is subgraph of G and it can dominate all vertices of $V(G)$, thus $\gamma_c(G) = t + 1$. ∎

Theorem 2. Let $G = V_{\{v\}}^1 \{F_{n_1}, F_{n_2}, \dots, F_{n_t}\}$ is amalgamation vetex of frienship graph, then
$$\gamma_c(G) = \begin{cases} 1, & v \text{ is centre vertex of } F_{n_i}, \ 2 \le i \le t \\ t + 1, & v \text{ is any vetex of degree } 2, \ 2 \le i \le t \end{cases}$$
Proof: If the linkage vertex of $G = V_{\{v\}}^1 \{F_{n_1}, F_{n_2}, \dots, F_{n_t}\}$ is centre vertex of F_{n_i} then for any $k \in V(G)\backslash v$ show that $d(v, k) = 1$. It means that v can dominate all vertices of $V(G)$, thus $\gamma_c(G) = 1$. If the linkage vertex of $G = V_{\{v\}}^1 \{F_{n_1}, F_{n_2}, \dots, F_{n_t}\}$ is any vetex of degree 2 of F_{n_i} then for any k element of $V(G)\backslash v$ show that $d(v, k) \ge 2$. Let S is connected dominating number of G, so we can take v_i which the centre vertex of every F_{n_i} with $d(v, v_i) = 1$ as the element of S. Then the set $S = \{v, v_i | 2 \le i \le t\}$ is subgraph of G and it can dominate all vertices of $V(G)$, thus $\gamma_c(G) = t + 1$. ∎

Theorem 3. Let $G = V_{\{v\}}^1 \{C_n, P_m, S_k, B_{x,y}\}$ is amalgamation vertex of cycle, path, star, and complete bipartite graph with the each order is $n \ge 3$, $m \ge 2$, $k \ge 3$, $x, y \ge 2$, then
$$\gamma_c(G) = \begin{cases} n + m - 2, & v \text{ of } S_k \text{ is one of the pendant}, v \text{ of } P_m \text{ is } v_1 \text{ or } v_m \\ n + m - 3, & v \text{ of } S_k \text{ is one of the pendant}, v \text{ of } P_m \text{ is } v_i \text{ for } 2 \le i \le n-1 \end{cases}$$
Proof: A cycle graph has a connected dominating number that is the number of vertices minus 2 (Theorem 4). The connected dominating vertex in a cycle graph are all vertices except two neighboring vertices, in this case all vertices can be chosen to be the connected dominating vertex. In a path graph, the connected dominating vertex are all vertices of degree 2. Thus, there are vertices that cannot be a dominating vertex, namely a vertex of degree 1. However, for the first case, one of the vertex whose the degree 1 in path is linkage vertex for vertex amalgamation. The star graph has a connected dominating number equal to one and one of its pendant is a linkage vertex. Meanwhile, in a complete bipartite graph, the connected dominating number is equal to two, i.e. two adjacent vertices. One of these vertex is the linkage vertex. Therefore connected dominating number of $G = V_{\{v\}}^1 \{C_n, P_m, S_k, B_{x,y}\}$ is the sum of each connected dominating number in every graph except a complete bipartite graph because an element of connected dominating set set is a linkage vertex, then
$\gamma_c = \gamma_c(C_n) + \gamma_c(P_m) + \gamma_c(S_k) + (\gamma_c(B_{x,y}) - 1)$
$\gamma_c = (n - 2) + (m - 2) + 1 + (2 - 1)$
$\gamma_c = n + m - 4 + 2$
$\gamma_c = n + m - 2$
While for the second case, the difference is that the linkage vertex of the graph P_m is a vertex of degree 2, then reduce one vertex of connected dominating number P_m. Therefore
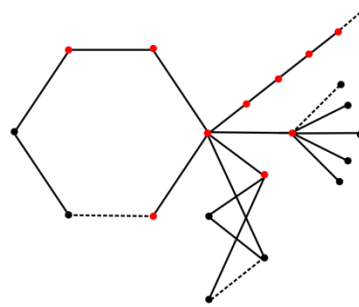$\gamma_c = \gamma_c(C_n) + (\gamma_c(P_m) - 1) + \gamma_c(S_k) + (\gamma_c(B_{x,y}) - 1)$
$\gamma_c = (n - 2) + ((m - 2) - 1) + 1 + (2 - 1)$
$\gamma_c = n + m - 5 + 2$
$\qquad \gamma_c = n + m - 3$
This number is minimum because subtracting just one from the connected dominating vertex will cause the subgraph that is formed to be unconnected. ∎

As an example of using the theorem above, suppose graph G In Figure 3 is $G = V_{\{v\}}^1 \{C_6, P_6, S_7, B_{2,3}\}$, then it has $(6 + 6 - 2) = 10$ connected dominating number.

Figure 3. Amalgamation vertex of cycle, path, star, and complete bipartite graf.

Theorem 4. Let $G = \bigvee_{\{v\}}^1 \{C_n, P_m, S_k, B_{x,y}\}$ is amalgamation vertex of cycle, path, star, and complete bipartite graph with the each order is $n \geq 3$, $m \geq 2$, $k \geq 3$, $x, y \geq 2$, then

$$\gamma_c(G) = \begin{cases} n + m - 3, & v \text{ of } S_k \text{ is a vertex with order } n - 1, v \text{ of } P_m \text{ is } v_1 \text{ or } v_m \\ n + m - 4, & v \text{ of } S_k \text{ is a vertex with order } n - 1, v \text{ of } P_m \text{ is } v_i \text{ for } 2 \leq i \leq n - 1 \end{cases}$$

Proof: By the same procedure to the proof in Theorem 15, we can start by observe that a cycle graph with order n has a connected dominating number equals $n - 2$ (Theorem 4). The connected dominating vertex in a cycle graph are all vertices except two neighboring vertices, in this case all vertices can be chosen to be the connected dominating vertex. In a path graph, the connected dominating vertex are all vertices of degree 2. Thus, there are vertices that cannot be a dominating vertex, namely a vertex of degree 1. However, for the first case, one of the vertex whose the degree 1 in path is linkage vertex for vertex amalgamation. The star graph has a connected dominating number equal to one and it is the centre vertex or vertex whose the degree is $k - 1$. It is also a linkage vertex in $= \bigvee_{\{v\}}^1 \{C_n, P_m, S_k, B_{x,y}\}$. Meanwhile, in a complete bipartite graph, the connected dominating number is equal to two, i.e. two adjacent vertices. One of these vertex is the linkage vertex. Therefore connected dominating number of $G = \bigvee_{\{v\}}^1 \{C_n, P_m, S_k, B_{x,y}\}$ is the sum of each connected dominating number in every graph except star and complete bipartite graph because an element of connected dominating set set is a linkage vertex, then

$\gamma_c = \gamma_c(C_n) + \gamma_c(P_m) + (\gamma_c(S_k) - 1) + (\gamma_c(B_{x,y}) - 1)$

$\gamma_c = (n - 2) + (m - 2) + (1 - 1) + (2 - 1)$

$\gamma_c = n + m - 4 + 1$

$\gamma_c = n + m - 3$

While for the second case, the difference is that the linkage vertex of the graph P_m is a vertex of degree 2, then reduce one vertex of connected dominating number P_m. Therefore

$\gamma_c = \gamma_c(C_n) + (\gamma_c(P_m) - 1) + (\gamma_c(S_k) - 1) + (\gamma_c(B_{x,y}) - 1)$

$\gamma_c = (n - 2) + ((m - 2) - 1) + (1 - 1) + (2 - 1)$

$\gamma_c = n + m - 5 + 1$

$\quad \gamma_c = n + m - 4$

This number is minimum because subtracting just one from the connected dominating vertex will cause the subgraph that is formed to be unconnected.   ∎

The last result is the case study of determination of bulog regional sub-divisions in east java using connected domination number theory. In this paper we only choose 18 vertex and 20 edes. Based on the location in the Figure 4, we represent the map into Bulog Office Graph or it call $BU - graph$ like in Figure 5. Based on the theory of connected domination number and the result of the observation of $BU - graph$, we get 6 vetices which make a subgraph of $BU - graph$ and can dominate all vertices in $BU - graph$. Thus. $\gamma_c(BU - graph) = 6$.

Figure 4. Map of bulog office in East Java.
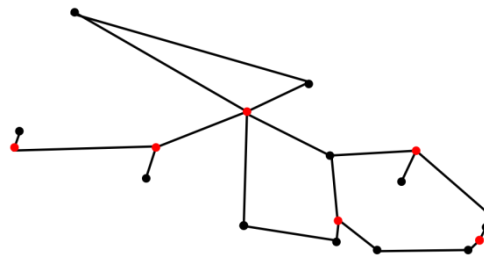
Figure 5. Representation of connected dominating set of bulog office graph.

3. Conclusion

In this paper, we can determine the connected domination number of unicyclic graph and some vertex amalgamation graphs which consisting of wheel, friendship, cycle, path, star, and complete bipartite graphs with some possibilities for the position of linkage vertex amalgamation. By apllying the theory, we also can determine the number of Bulog Regional Sub-divisions in east Java. From 18 location, we get 6 location can be recommended as Bulog Regional Sub-divisions to dominate all location.

Open problem 1. Find the connected domination number for another particular classes of graphs and the graphs obtained from graph operations such as edge amalgamation, join, shackle, etc.

Open problem 2. Find the algorithm to solve the connected domination number for case in real life.

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