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1. Judul Jurnal Ilmiah : *Dominating Number of Distance Two of Cororna Products of Graphs*
2. Penulis Jurnal Ilmiah : 1.Reni Umilasari, S.Pd, M.Si
   2. Dr. Darmaji, MT.
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Dominating number of distance two of corona product of graphs

by Reni Umilasari

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Dominating number of distance two of corona product of graphs

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Abstract

Dominating set S in graph $G = (V, E)$ is a subset of $V(G)$ such that every vertex of G which is not element of S are connected and have distance one to S. Minimum cardinality among dominating sets in a graph G is called dominating number of graph G and denoted by $\gamma(G)$. While dominating set of distance two which denoted by S_2 is a subset of $V(G)$ such that every vertex of G which is not element of S are connected and have maximum distance two to S_2. Dominating number of distance two $\gamma_2(G)$ is minimum cardinality of dominating set of distance two S_2. The corona $G \odot H$ of two graphs G and H where G has p vertices and q edges is defined as the graph G obtained by taking one copy of G and p copies of H, and then joining by an edge the $i-th$ vertex of G to every vertex in the $i-th$ copy of H. In this paper, we determine the dominating number of distance two of paths and cycles. We also determine the dominating number of distance two of corona product of path and any graphs as well as cycle and any graphs.

Keywords: cycle, path, corona product, dominating number, dominating set
Mathematics Subject Classification : 05C69

1. Introduction

Dominating set S in a graph $G = (V, E)$ is a subset of $V(G)$ such that every vertex of G which is not element of S is connected and have distance one to S. The minimum cardinality among

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dominating sets in a graph G is called dominating number of a graph G and denoted by $\gamma(G)$. Therefore, dominating number very closely related to dominating set.

The research on dominating set in a graph have been started in 1850 when the European chess players want to solve "dominating queens" problems. In this problem, dominating set is used to determine number of queens such that every square on a standard 8×8 chessboard is either occupied by a queen or can be directly attacked by a queen [9]. Research on dominating set and dominating number of a graph have been conducted such as c-dominating sets for families of graphs [9], dominating cartesian product of cycles [8], and total domination number of grid graph [4].

There are some applications of dominating set and dominating number of graph, i.e finding the school bus route and numbers of the bus stop such that students should not go too far from their home [7], finding number of fire engine that should be put in crossroads to cover all the region in case of fire and emergency first aid stations. Suppose that a natural disaster has struck some region consisting of many small villages. The vertices of a graph represent the villages in the region. An edge joining two vertices indicates that an emergency first aid station set up in one of the corresponding villages can also serve the other one. Then, a minimum dominating set of the graph would prescribe a way of serving the entire region with a minimum number of first aid stations [5]. But if the number of the emergency first aid stations is limited, it can be minimized by assuming one emergency first aid stations can dominate two nearest villages. This motivate us to define dominating number of distance two. Graphs that will be considered are path and cycle which are operated by corona product with any graphs. The corona $G \odot H$ of two graphs G and H where G has p vertices and q edge is defined as the graph G obtained by taking one copy of G and p copies of H, and then joining by an edge the $i-th$ point of G to every point in the $i-th$ copy of H [6].

Dominating set S in a graph $G = (V, E)$ is a subset of $V(G)$ such that each vertex of G which is not an element of S is connected and have distance one to S. The minimum cardinality among dominating sets in a graph G is called dominating number of a graph G and denoted by $\gamma(G)$ [7]. While dominating set of distance two S_2 is a subset of $V(G)$ such that each vertex G which is not an element of S_2 have distance less than or equals two to S_2. Dominating number of distance two, is denoted by $\gamma_2(G)$, is the minimum cardinality of dominating set of distance two. Figure 1 shows the minimum dominating sets of distance one and distance two, respectively. In Figure 1. (a) vertices v_2 and v_5 represent the vertices which are the element of the minimum dominating set of distance one, while vertex u_7 in figure 1 (b) represents the vertex which is the element of the minimum dominating set of distance two.

2. Main Result

We begin this section with the following theorem regarding to the dominating number of distance two of path and cycle.

Theorem 2.1. *Path on m vertices P_m for any $m \geq 2$ has dominating number of distance two* $\gamma_2(P_m) = \lceil \frac{m}{5} \rceil$.

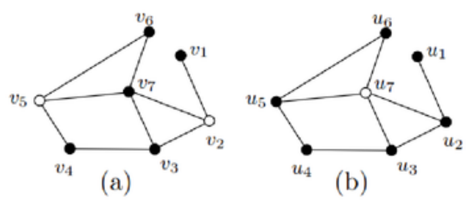

Figure 1. (a) Minimum dominating set of distance one (b) Minimum dominating set of distance two

Proof. Let P_m be the Path with order m. Because the maximal degree of every vertex on P_m is equal 2, then one vertex can dominate at most 5 vertices with the distance less than or equal 2. Therefore the minimal number of vertices that can dominate by m vertices are $\lceil \frac{m}{5} \rceil$. Thus, $\gamma_2(P_m) \geq \lceil \frac{m}{5} \rceil$.

We now show that $\lceil \frac{m}{5} \rceil$ is the minimal number element of S_2. Suppose $\gamma_2(P_m) = \lceil \frac{m}{5} \rceil - 1$, then the maximal number of vertices that can be dominated on distance at most two are $5(\lceil \frac{m}{5} \rceil - 1) \leq 5(\frac{m+4}{5} - 1) = m - 1$. Then, there are only $m - 1$ vertices that can be dominated or at least one vertex of $V(P_m)$ cannot be dominated. It means that $|S_2| = \gamma_2(P_m) \neq \lceil \frac{m}{5} \rceil - 1$.

Because $\lceil \frac{m}{5} \rceil$ is minimal number of vertices that can dominate all vertices on P_m, then $\gamma_2(P_m) = \lceil \frac{m}{5} \rceil$. □

Theorem 2.2. *Cycle on n vertices C_n for any $n \geq 3$ has dominating number of distance two $\gamma_2(C_n) = \lceil \frac{n}{5} \rceil$.*

Proof. Cycle is regular 2 graph. Then for every u_i element of $V(C_n)$, $deg(u_i) = 2$. Because for every u element S_2, u can dominates at most 5 vertices with the distance less than or equal two and $|C_n| = n$, thus $|S_2| \geq \lceil \frac{n}{5} \rceil$.

To show that $\lceil \frac{n}{5} \rceil$ is the minimal number element of S_2, we suppose $\gamma_2(C_n) = \lceil \frac{n}{5} \rceil - 1$. Then the maximal number of vertices that can be dominate on distance at most two are $5(\lceil \frac{n}{5} \rceil - 1) \leq 5(\frac{n+4}{5} - 1) = n - 1$. It implies that only $n - 1$ vertices that can be dominated by S_2 or at least one vertex of C_n cannot be dominated. Therefore, $|s_2| = \gamma_2(C_n) \neq \lceil \frac{n}{5} \rceil - 1$.

We can conclude that $\lceil \frac{n}{5} \rceil$ is minimal number of vertices that can dominate all vertices on C_n, then $\gamma_2(C_n) = \lceil \frac{n}{5} \rceil$. □

For dominating number of distance two of corona product graphs will be begun with the following observations regarding to the relationship between a graph G with the diameter at most 2 and its dominating number of distance two.

Observation 2.1. *If a graph G has diameter at most 2, then* $\gamma_2(G) = 1$

Since the diameter of graph G is at most 2, then the distance of any two vertices in G is at most 2. By taking any $v_i \in V(G)$ as vertex element of dominating set of distance two $S_2(G)$, it implies that $d(v_i, V(G)) \leq 2$. Therefore, $\gamma_2(G) = 1$.

Some graphs which have diameter less than or equals two such as Complete K_n, Wheel W_n, Fan F_n, Friendship W_2^m, and Star S_n. Thus, $\gamma_2(K_n) = \gamma_2(W_n) = \gamma_2(F_n) = \gamma_2(W_2^m) = \gamma_2(S_n) = 1$.

Observation 2.2. *Let G and H be connected graphs of order m and n, respectively. If diameter of G is one $\gamma_2(G_n \odot H_m) = 1$.*

Because G_m and H_n are corona product graphs, then $\forall v_i \in G_m$ and $v_{i,j} \in H_i$ imply $d(v_i, v_{i,j}) = 1$. Take arbitrary $v_k \in V(G_m)$ for $v_k \neq v_i$ as the vertex element of dominating set of distance two which implies $d(v_k, v_i) = 1$, because the diameter of G_m is equal 1. While $d(v_k, v_{i,j}) = d(v_k, v_i) + d(v_i, v_{i,j}) = 2$ or any vertex in G has maximal distance 2 to all vertices element $V(G_m \odot H_n)$, then $\gamma_2(G_m \odot H_n) = 1$.

Graph with the diameter equals one for example *Complete* K_n. Then $\gamma_2(K_n \odot H) = 1$, where H is any graph.

We now present the dominating number of distance two of corona product of path with any graph and cycle with any graph as follows.

Theorem 2.3. *Let $P_m \odot G_n$ be a corona product of path P_m and any graphs G_n. Then dominating number of distance two $\gamma_2(P_m \odot G_n) = \lceil \frac{m}{3} \rceil$ for $m \geq 2$.*

Proof. Suppose $V(P_m \odot G_n) = \{v_1, v_2, \ldots, v_m\} \cup \{v_{i,j} | 1 \leq i \leq m, 1 \leq j \leq n\}$, then $|P_m \odot G_n| = mn + m$. The cases below show three posibilities of minimum number of vertices which are the element of dominating number of distance two on $P_m \odot G_n$.

Case 1: $S_2 \in V(G_i)$
For every $v_{i,j}$ element of $V(G_i)$, $v_{i,j}$ can dominate at most $n + 3$ vertices. Then the maximal number of vertices which are the element of dominating number of distance two is $\frac{mn+m}{n+3}$, thus $|S_2| \leq \frac{mn+m}{n+3}$.

Case 2: $S_2 \in V(P_m)$
For every v_i element of $V(P_m)$, v_i can dominate at most $3n + 5$ vertices. Then the maximal number of vertices which are the element of dominating number of distance two is $\frac{mn+m}{3n+5}$, thus $|S_2| \leq \frac{mn+m}{3n+5}$.

Case 3: $S_2 \in V(G_i) \cup V(P_m)$
For every v_i element of $V(P_m)$ and $v_{i,j}$ element of $V(G_i)$, then two vertices can dominate at most $(n + 3) + (3n + 5)$ vertices. Thus $|S_2| \leq \frac{2(mn+m)}{(n+3)+(3n+5)} = \frac{mn+m}{n+4}$.

By three cases above $\frac{mn+m}{3n+5} \leq \frac{mn+m}{n+4} \leq \frac{mn+m}{n+3}$, then we can take $V(P_m)$ as the vertex element of S_2. Because $\frac{mn+m}{3n+5} = \frac{m(n+1)}{3(n+1)+2} < \frac{m(n+1)}{3(n+1)} = \frac{m}{3}$, then the interval or distance of every vertex element of S_2 is equal 3. Because $|S_2|$ must be integer number, then minimal number of vertices which be the element of dominating number of distance two of $V(P_m \odot G_n)$ are $\lceil \frac{m}{3} \rceil$. Therefore $\gamma_2(P_m \odot G_n) \geq \lceil \frac{m}{3} \rceil$

Next, we show that $\lceil \frac{m}{3} \rceil$ is the minimal number element of S_2. Let $|S_2| = \lceil \frac{m}{3} \rceil - 1$, then the maximal number of vertices which are the element of S_2 is

$$\begin{aligned}\left(\left\lceil \frac{m}{3} \right\rceil - 1\right)(3n+3) &\leq \left(\frac{m+2}{3} - 1\right)(3n+3) \\ &= mn + m - n - 1 \\ &< mn + m.\end{aligned}$$

It can conclude that not all vertices can be dominated. Therefore, $|S_2| \neq \lceil \frac{m}{3} \rceil - 1$ and $\lceil \frac{m}{3} \rceil$ is minimum dominating number of $P_m \odot G_n$. Thus it is proven that $\gamma_2(P_m \odot G_n) = \lceil \frac{m}{3} \rceil$. □

Theorem 2.4. *Let $C_n \odot H_m$ be a corona product of cycle C_n and any graphs H_m. Then dominating number of distance two $\gamma_2(C_n \odot H_m) = \lceil \frac{n}{3} \rceil$ for $n \geq 3$.*

Proof. Let $V(C_n \odot H_m) = \{u_1, u_2, \ldots, u_n\} \cup \{u_{i,j} | 1 \leq i \leq n, 1 \leq j \leq m\}$, then $|C_n \odot H_m| = nm + n$. Some cases below show three posibilities of vertices which element of minimum dominating set of graph $C_n \odot H_m$.

Case 1: $S_2 \in V(H_i)$
For every $u_{i,j}$ element of $V(H_i)$, vertex $u_{i,j}$ can dominate at most $m + 3$ vertices. Then the maximal number of vertices which are the element of dominating number of distance two is $\frac{nm+n}{m+3}$, then $|S_2| \leq \frac{nm+n}{m+3}$.

Case 2: $S_2 \in V(C_n)$
For every u_i elemen $V(C_n)$, vertex u_i can dominate at most $3m + 5$ vertices. Then the maximal number of vertices which are the element of dominating number of distance two is $\frac{nm+n}{3m+5}$, then $|S_2| \leq \frac{nm+n}{3m+5}$.

Case 3: $S_2 \in V(H_i) \cup V(C_n)$
For every u_i element of $V(C_n)$ and $u_{i,j}$ element of $V(H_i)$, then two vertices can dominate at most $(m+3) + (3m+5)$ vertices, thus it implies $|S_2| \leq \frac{2(nm+n)}{(m+3)+(3m+5)} = \frac{nm+n}{m+4}$.

From case 1 up to 3, $\frac{nm+n}{3m+5} \leq \frac{nm+n}{m+4} \leq \frac{nm+n}{m+3}$. Then we can take the minimal number of S_2 are vertex set of Cycle C_n. Because $\frac{nm+n}{3m+5} = \frac{n(m+1)}{3(m+1)+2} < \frac{n(m+1)}{3(m+1)} = \frac{n}{3}$, then the interval or distance of every vertex element of S_2 is equal 3. We know that $|S_2|$ must be integer number and distance of every vertex element of S_2 is equal 3, then minimal number of vertices which are the element of dominating number of distance two of $V(C_n \odot H_m)$ is $\lceil \frac{n}{3} \rceil$, thus $\gamma_2(C_n \odot H_m) \leq \lceil \frac{n}{3} \rceil$.

Therefore, to show that $\lceil \frac{n}{3} \rceil$ is minimal dominating number, we can take $|S_2| = \lceil \frac{n}{3} \rceil - 1$. And it makes the maximal number of vertices which are the element of S_2 is

$$\begin{aligned}\left(\left\lceil \frac{n}{3} \right\rceil - 1\right)(3m+3) &\leq \left(\frac{n+2}{3} - 1\right)(3m+3) \\ &= nm + n - m - 1 \\ &< nm + n.\end{aligned}$$

We can conclude that not all vertices can be dominated. Therefore, $|S_2| \neq \lceil \frac{n}{3} \rceil - 1$ and $\lceil \frac{n}{3} \rceil$ is minimum dominating number of $C_n \odot H_m$. Then $\gamma_2(C_n \odot H_m) = \lceil \frac{n}{3} \rceil$. □

3. Conclusion

We conclude this paper with an open problem regarding to the dominating number of corona product of any two graphs G and H, that is

Open problem 4.1. For any graph G on m vertices and any graph H on n vertices, find $\gamma_2(G \odot H)$.

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